Tutorial:
Resource-efficient preparation of matrix product states with dynamic circuits

Kevin C. Smith
IBM Quantum

I. Introduction to MPS

1. What are they?

2. Why prepare them?

II. Techniques for preparing MPS on quantum processors

1. Unitary preparation — various flavors and current literature

2. Adaptive preparation (temporal compression)

3. Bonus: Holographic preparation (spatial compression)

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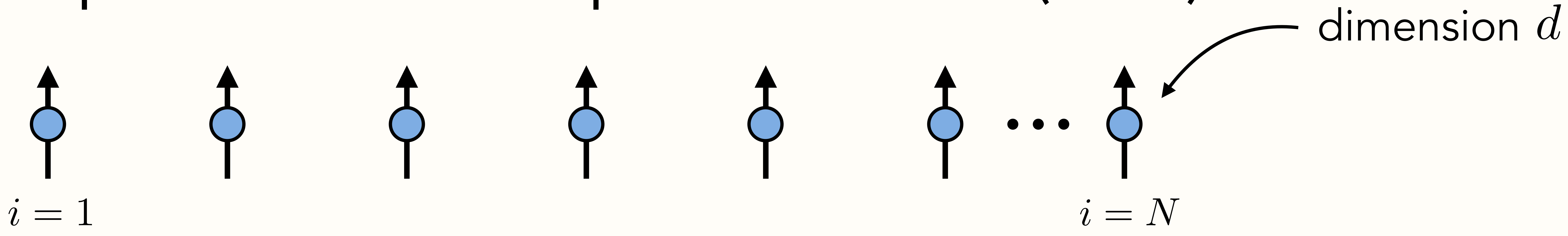
A primer on matrix product states (MPS)



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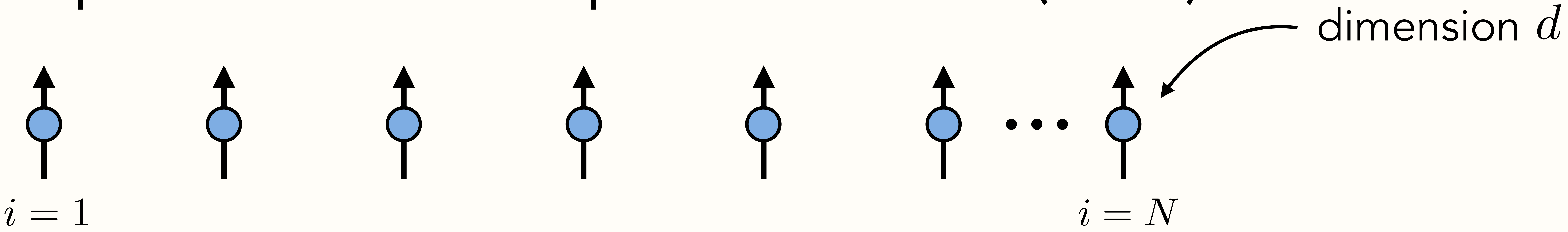

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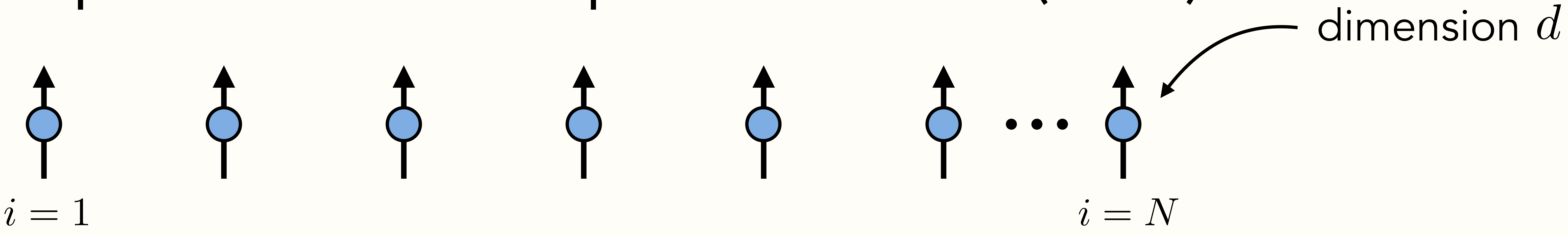

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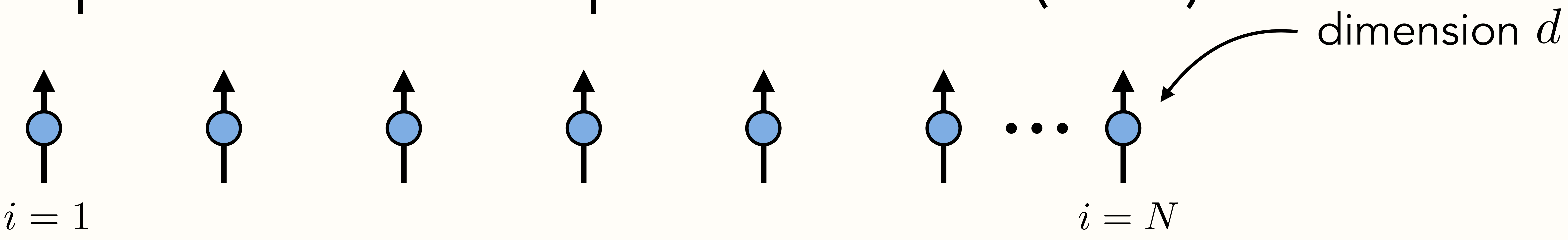

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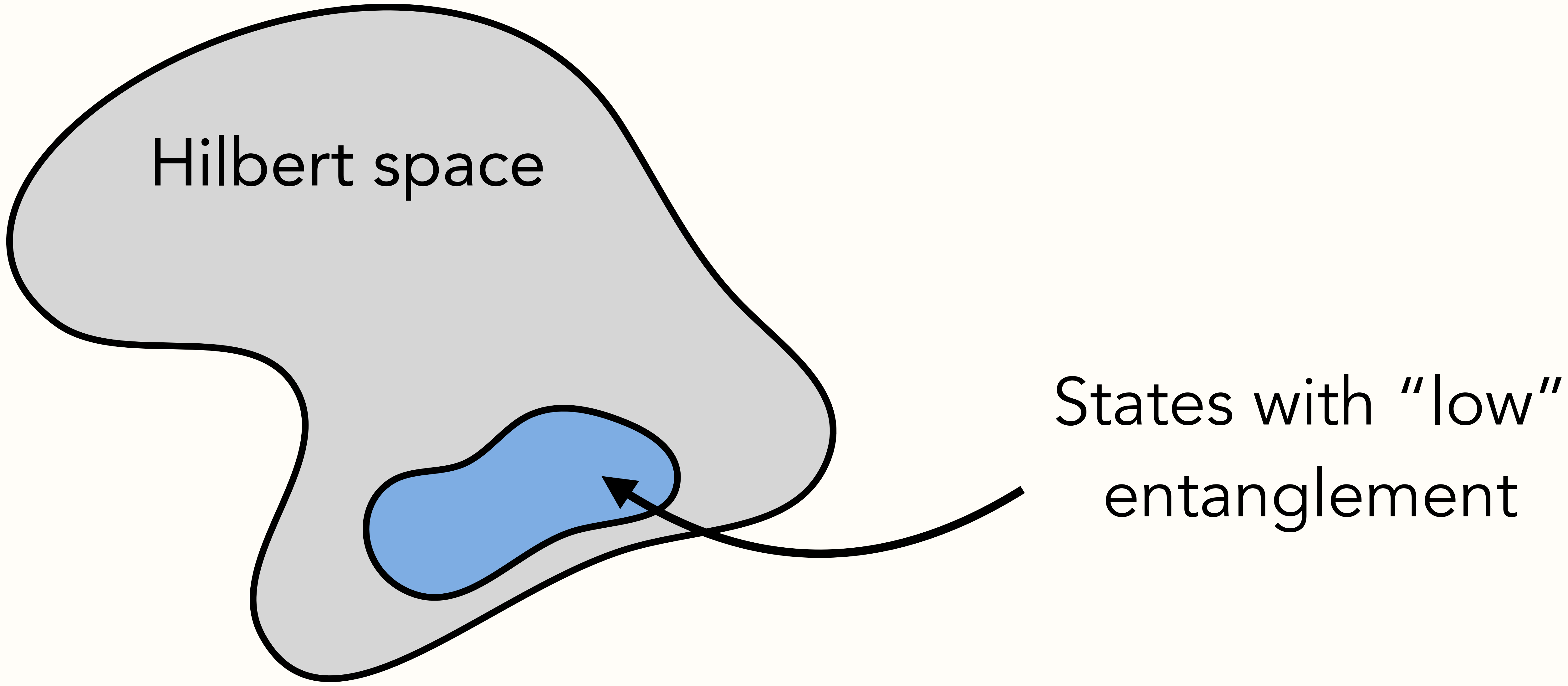

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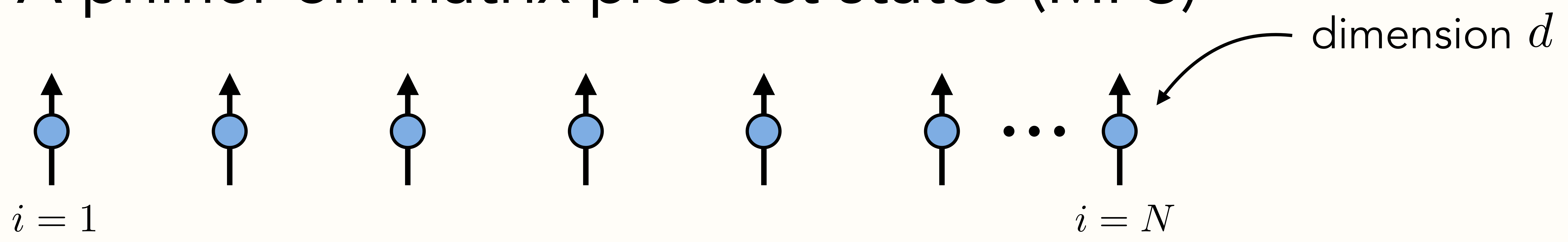

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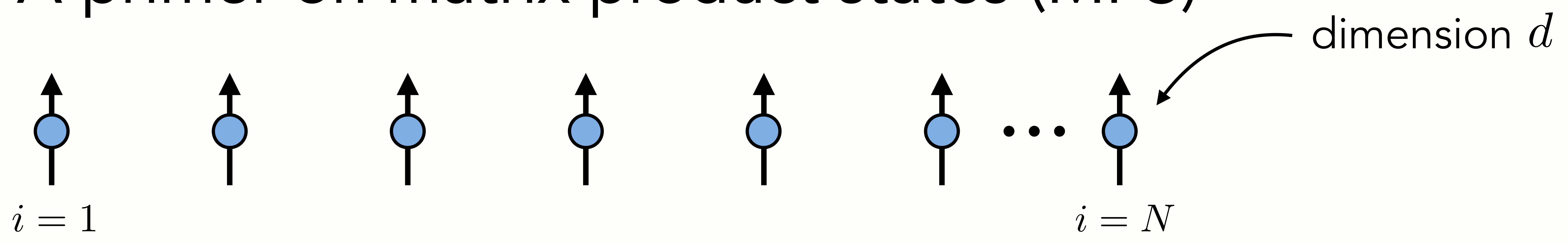

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$D \times D$ matrices

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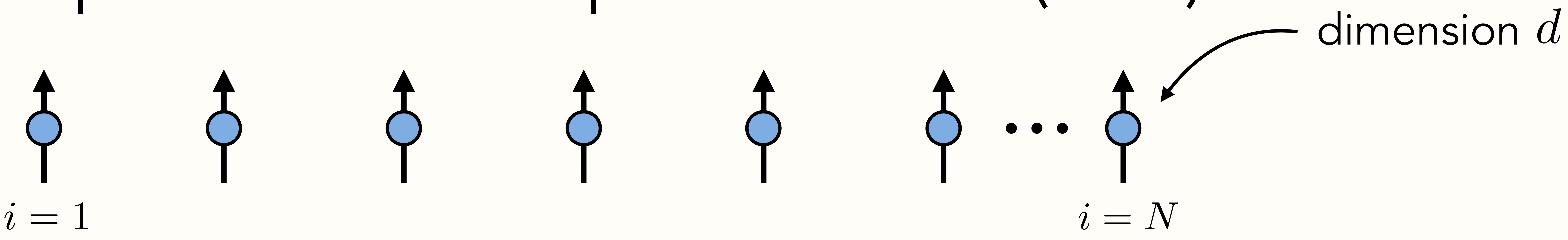

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Boundary conditions:

Periodic: $X \to I$

Open: $X \to |R\rangle\langle L|$

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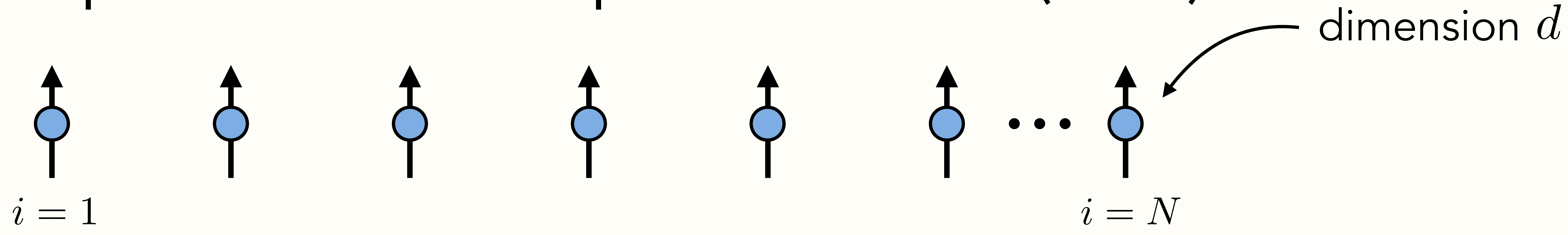

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Example: $\mathrm{Tr}(A^0 A^1 A^1 A^0 A^1 \ldots A^0)$ $|01101\ldots1\rangle$ (Periodic BC)

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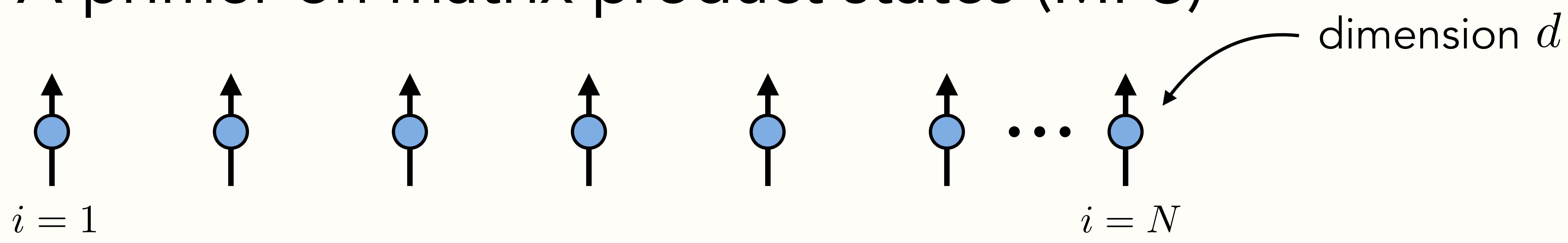

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$(N)dD^2$ complex coefficients

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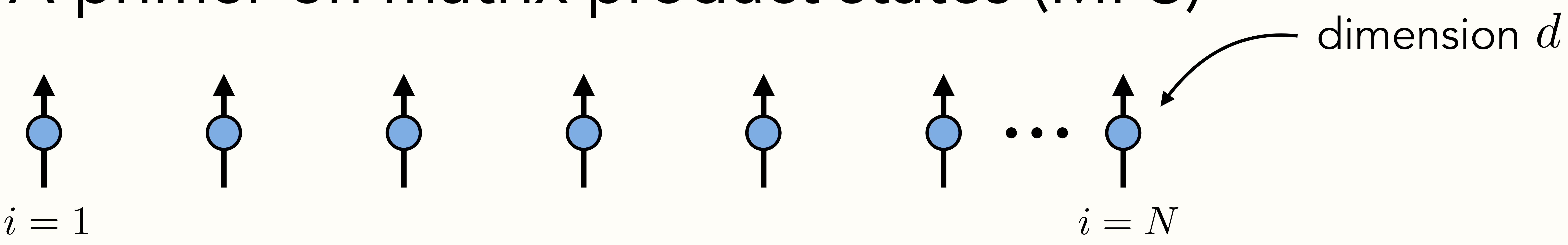

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Matrix product state with

bond (virtual) dimension D, physical dimension d

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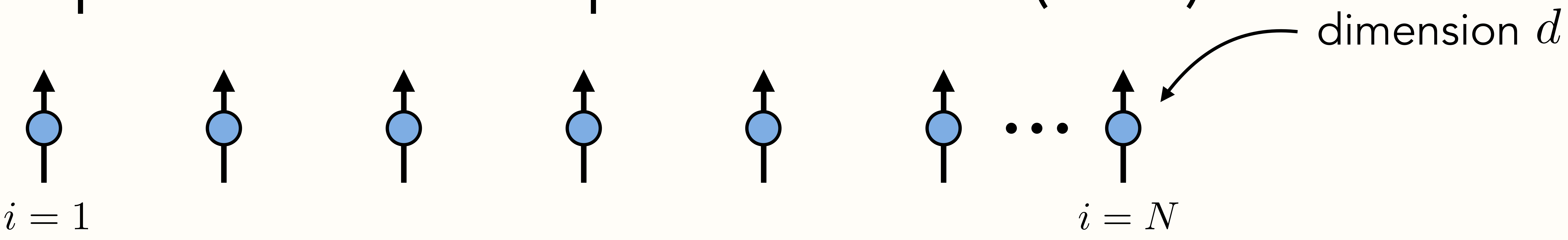

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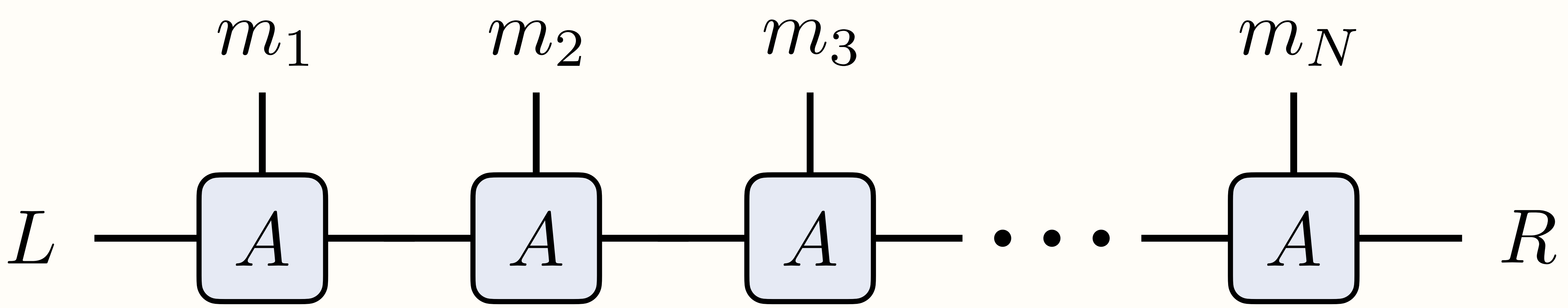

What states do MPS describe?

Example: the GHZ state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|000\ldots0\rangle + |111\ldots1\rangle) = \sum_{\vec{m}} \mathrm{Tr}(A^{m_1}A^{m_2}A^{m_3}\ldots A^{m_N})\,|m_1m_2m_3\ldots m_N\rangle$$

$$A^0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad A^1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

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More broadly:

- Ground state of gapped local 1D Hamiltonians
- Area law entanglement $S \sim O(1)$
- Exponentially decaying correlations

Hastings, Journal of Statistical Mechanics: Theory and Experiment (2007)

Some more resources on tensor networks

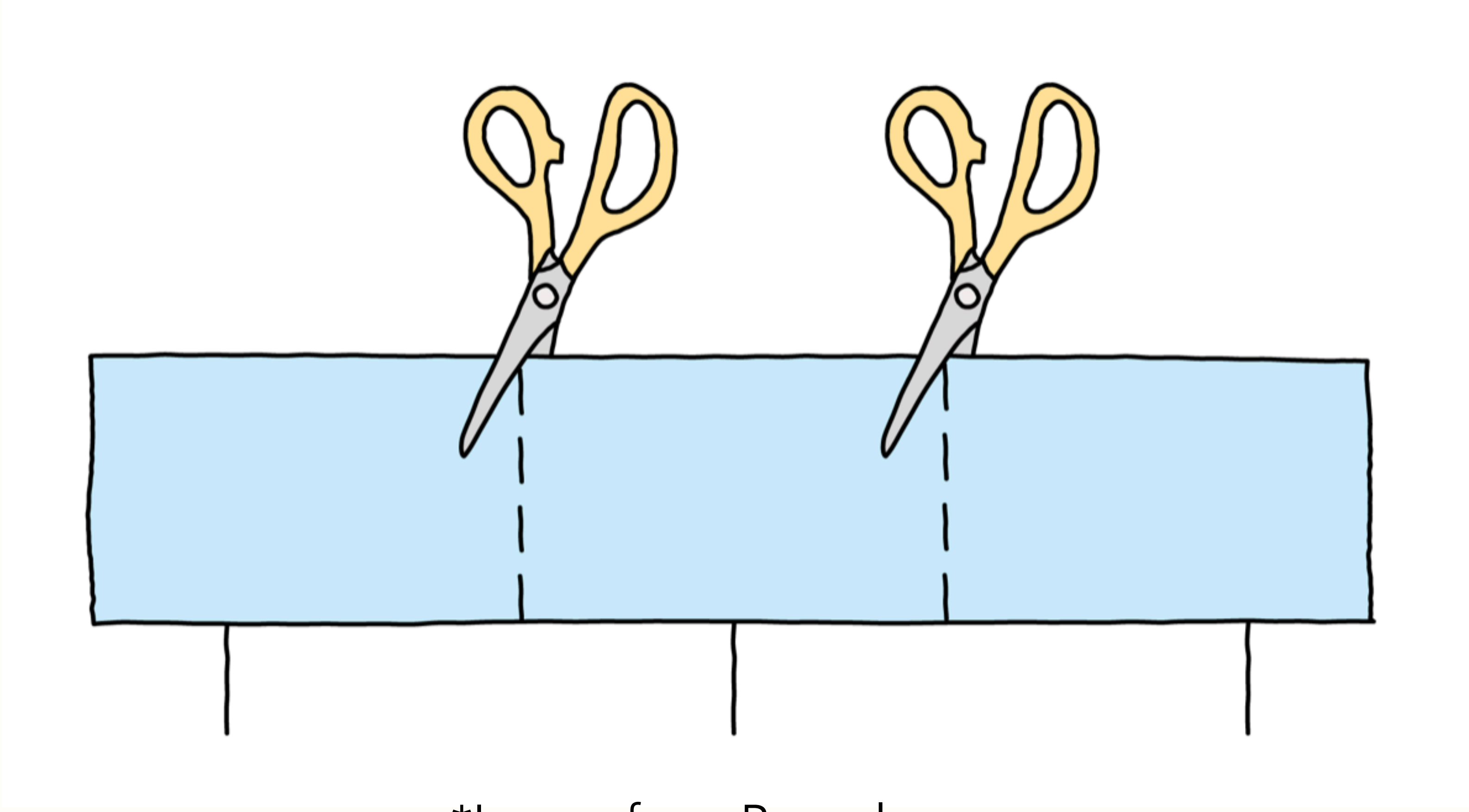


*Image from Pennylane

- PennyLane tutorial: "*Introducing matrix product states for quantum practitioners*" by Korbinian Kottmann

- Bridgeman and Chubb, "*Hand-waving and Interpretive Dance: An Introductory Course on Tensor Networks*", arXiv:1603.03039v4

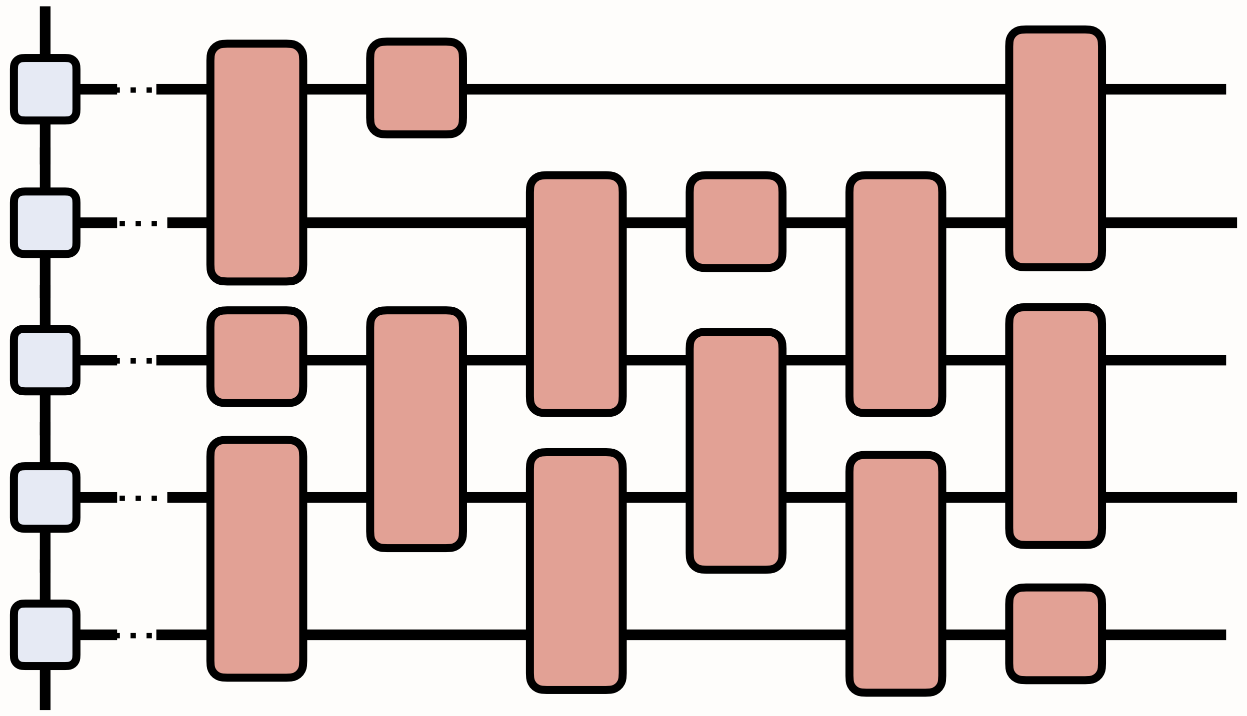
- Roman Orus, "*A Practical Introduction to Tensor Networks: Matrix Product States and Projected Entangled Pair States*", arXiv:1306.2164

Why prepare matrix product states?

Why prepare matrix product states?

Reason #1: They are a fundamentally important and useful class of states

- Classification of quantum phases [1, 2], many paradigmatic examples (GHZ, AKLT, Dicke,…)
- Prepare MPS → dynamics [3]; Example: many-body scarring [4]

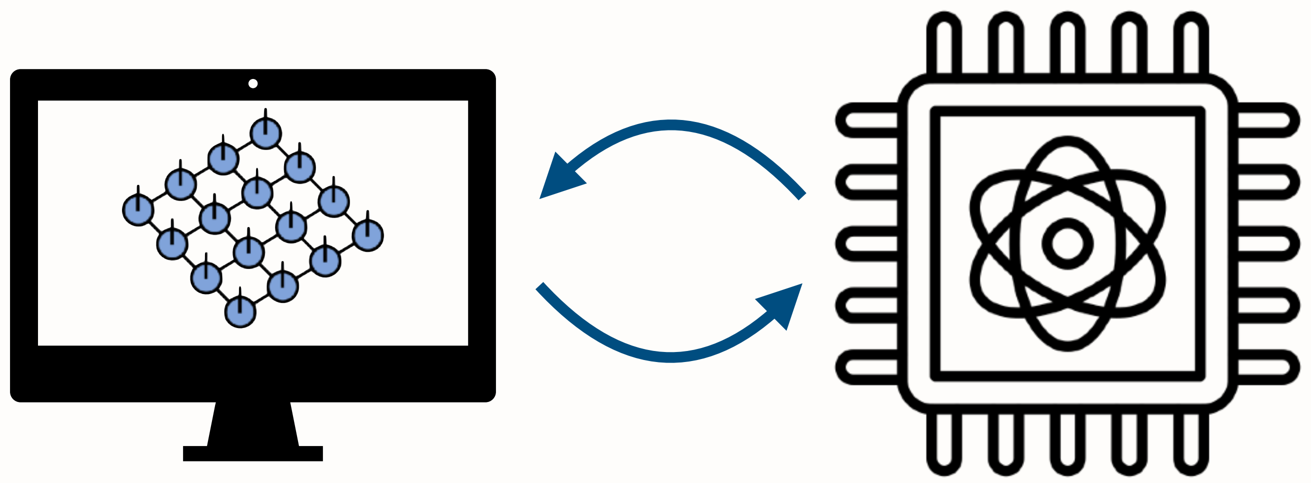


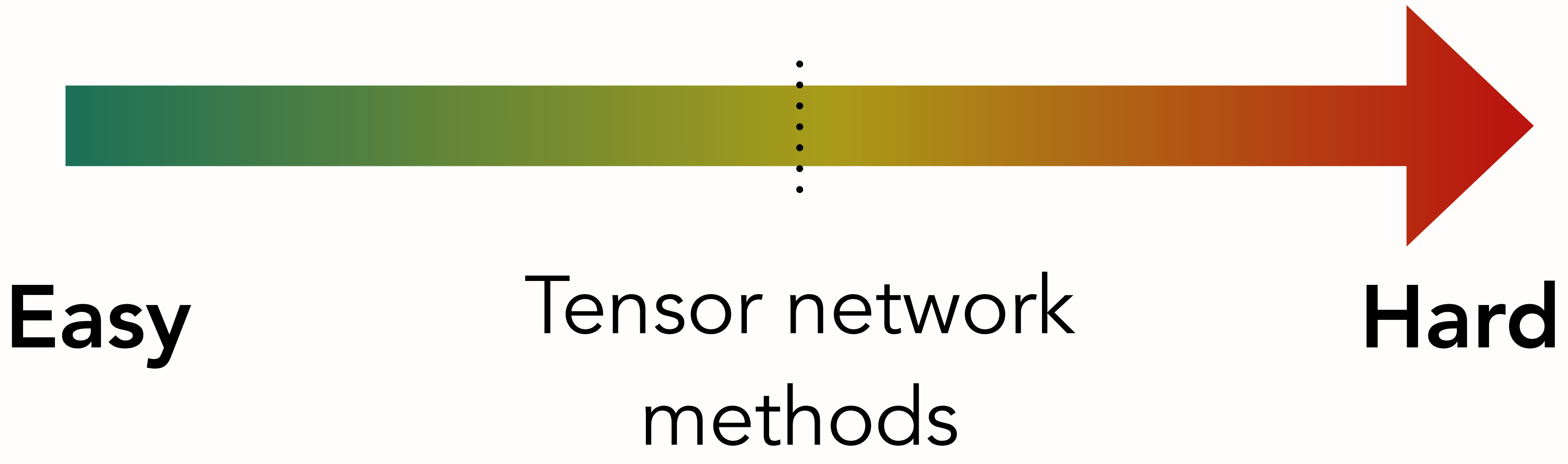
- Measurement-based quantum computation [5, 6]

[1] Cirac et al., Rev. Mod. Phys. (2021)
[2] Schuch et al., PRB (2011)
[3] Martin et al., arXiv:2305.19231 (2024)
[4] Gustafson et al., Quantum (2023)
[5] Gross et al., PRL (2007)
[6] Wang et al., PRA (2017)

Why prepare matrix product states?

Reason #2: They offer an interface between classical and quantum computing





- Pre-training variational algorithms [1, 2]
- Tensor network ansätze for quantum machine learning [3, 4]
- Loading classical data (e.g., images [5], probability distributions [6])
- Approximate quantum compilation [7]

[1] Rudolph et al., Nat. Comm. (2023)
[2] Khan et al., Nat. Comm. (2023)
[3] Rieser et al., PRSA (2023)
[4] Liu et al., PRL (2022)
[5] Jobst et al., arXiv: 2311.07666 (2023)
[6] Iaconis et al., npj Quantum Info (2024)
[7] Robertson et al., arXiv: 2301.08609 (2024)

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1. Unitary preparation — various flavors and current literature
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Unitary preparation of MPS

Target: $|\Psi\rangle = \sum_{\vec{m}} \langle L| A^{m_1} A^{m_2} A^{m_3} \dots A^{m_N} |R\rangle |m_1 m_2 m_3 \dots m_N\rangle$

(for simplicity, assume D=2, d=2)

C. Schön et al, PRL (2005)

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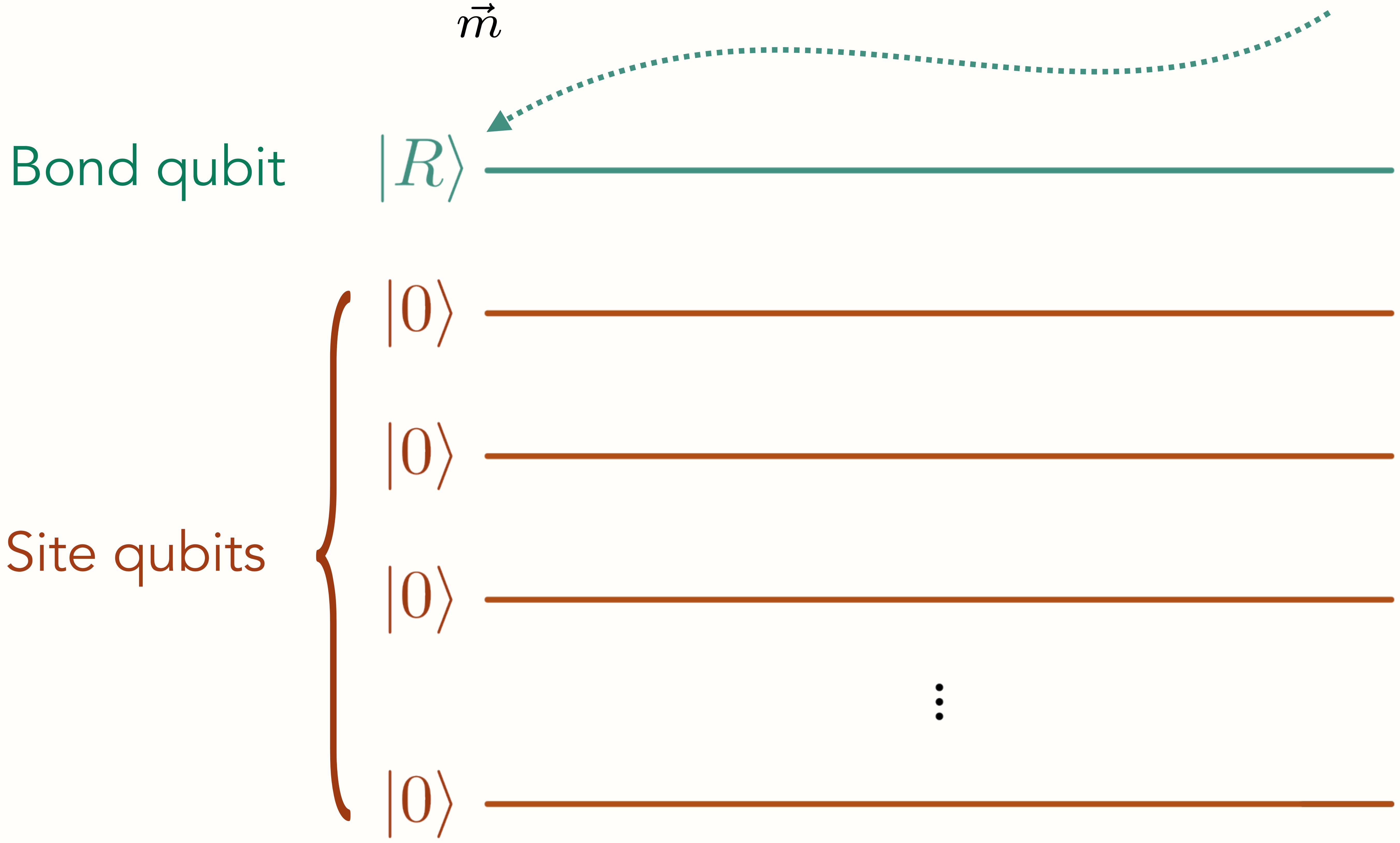

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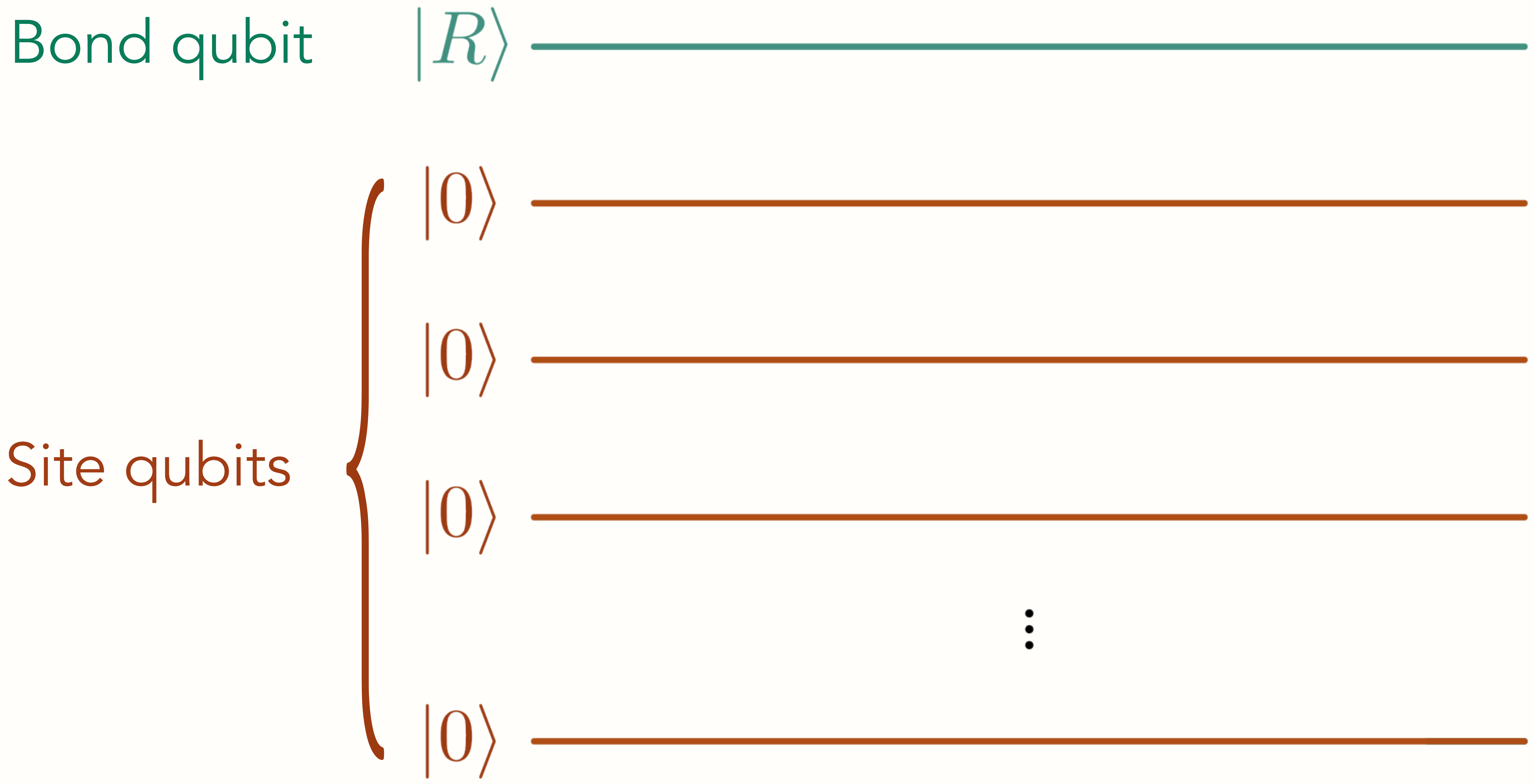

$$U = \sum_m A^m \otimes |m\rangle \langle 0| + C$$

Acts on bond qubit

Prepares site

Technical detail: Left-canonical form

$$\sum_m (A^m)^\dagger A^m = \mathbb{1}$$

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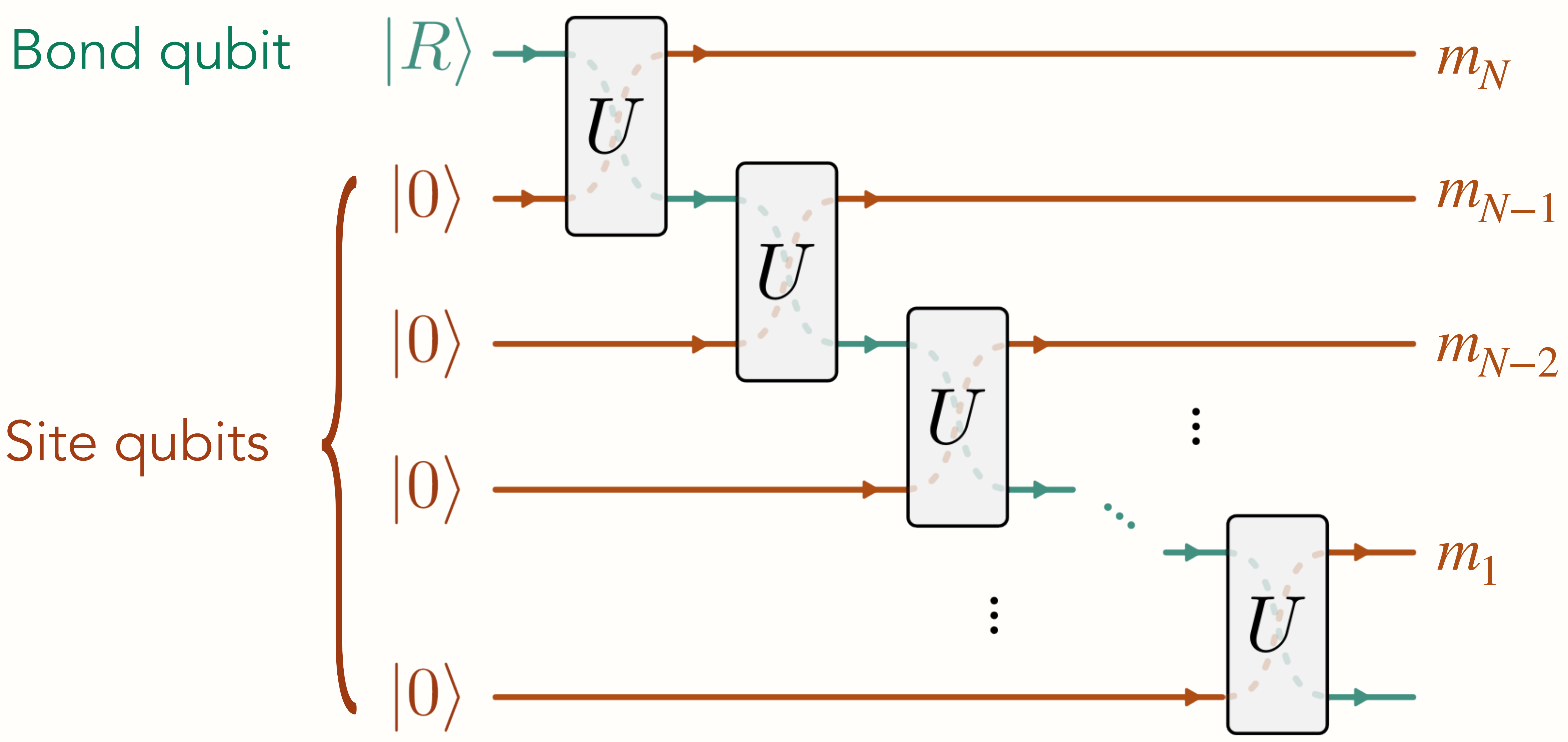

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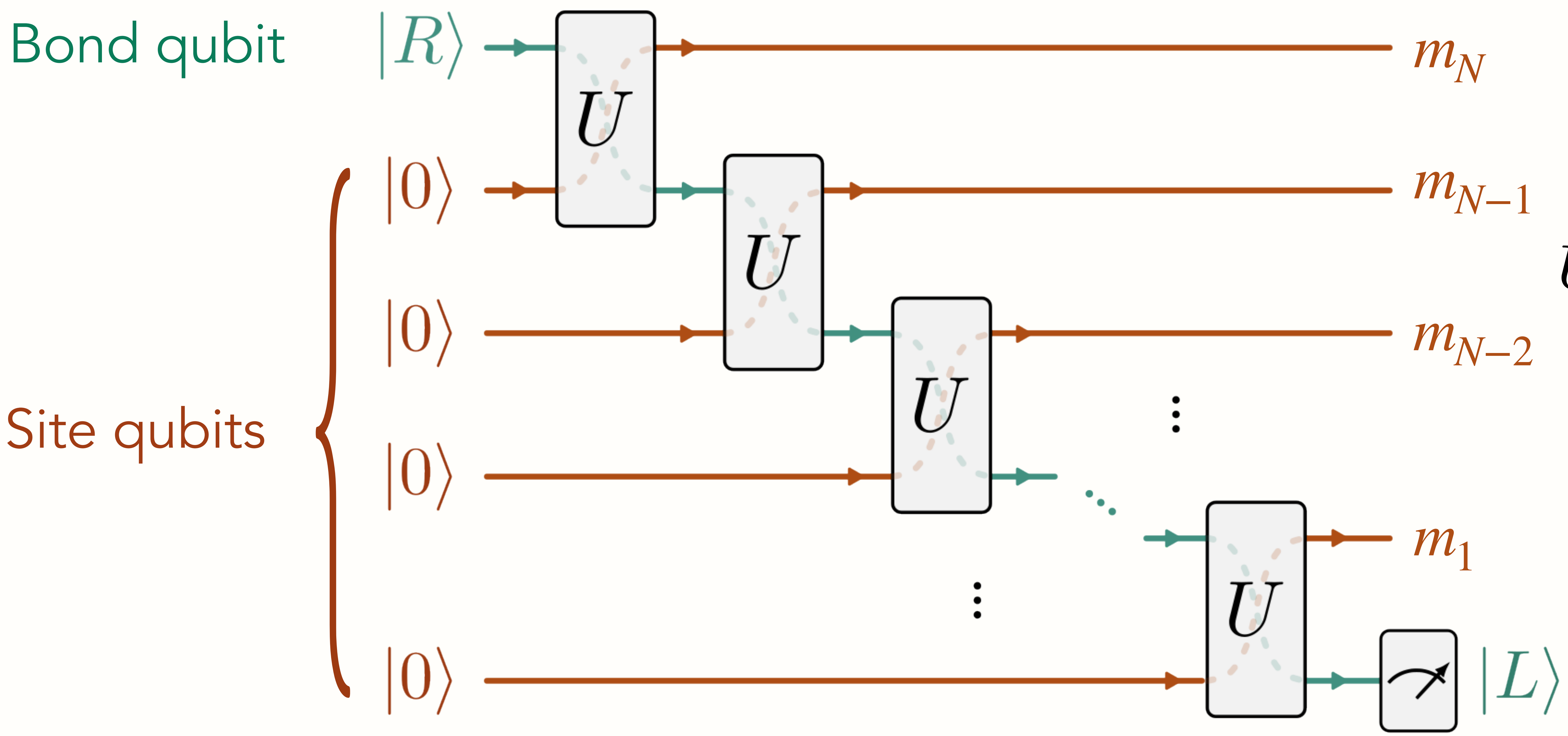

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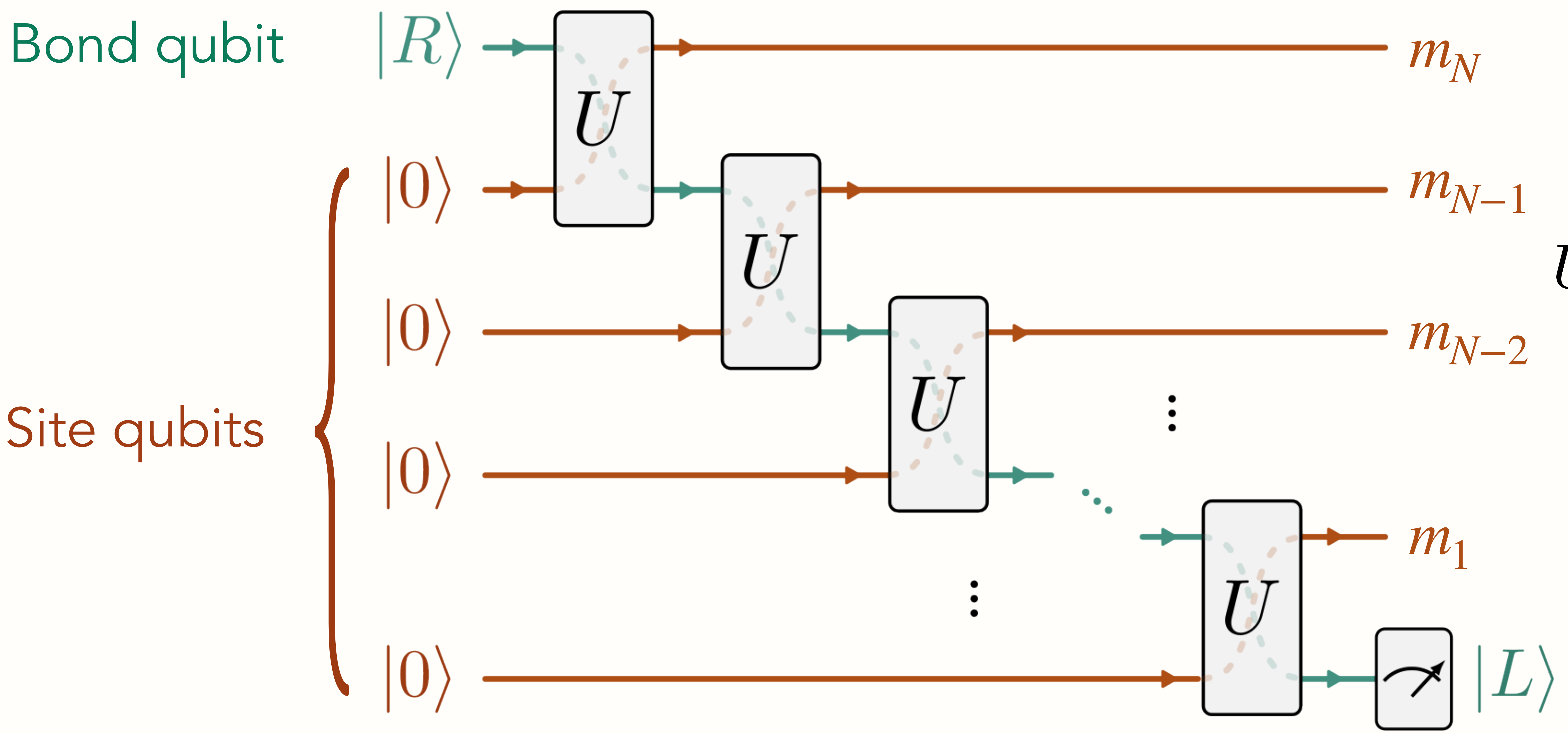

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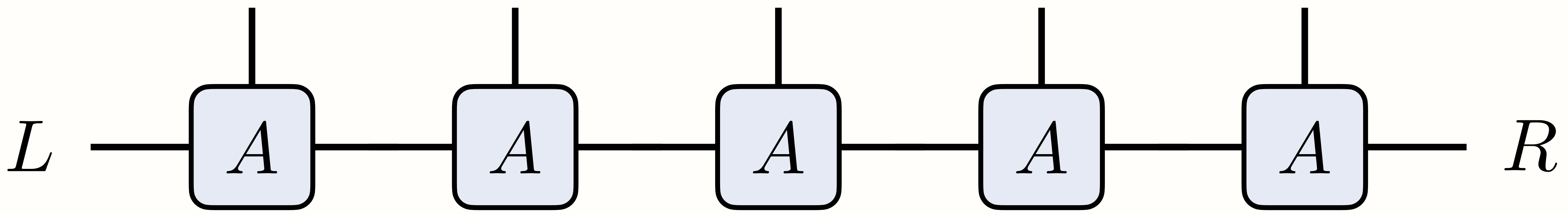
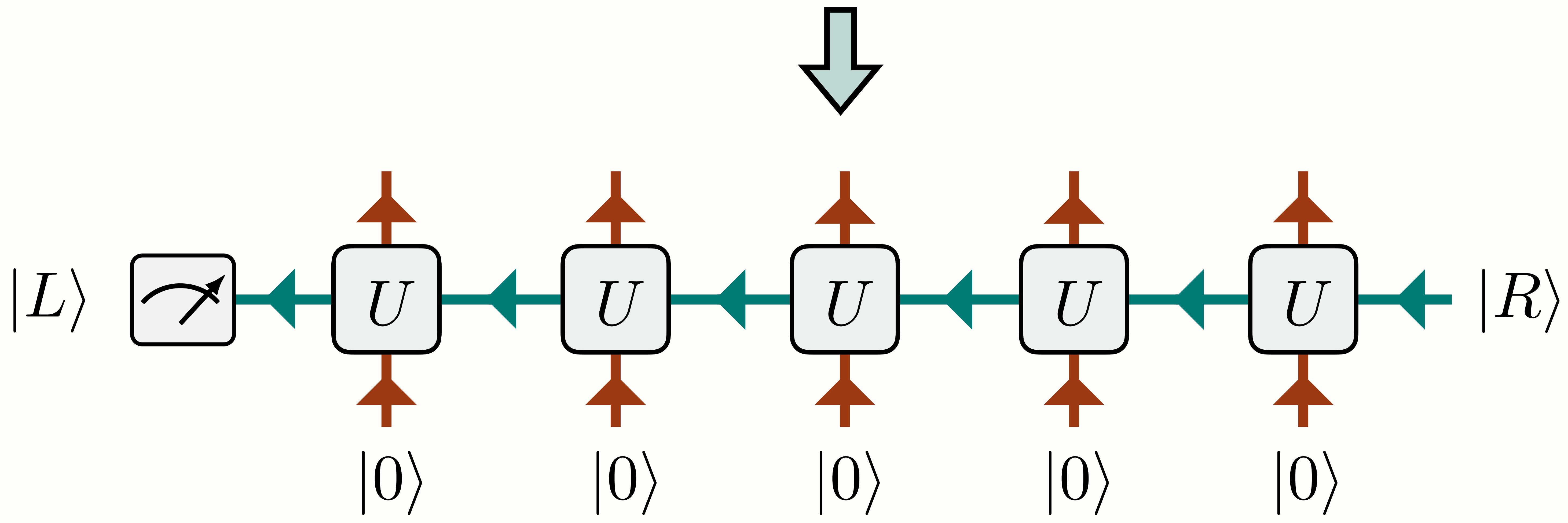
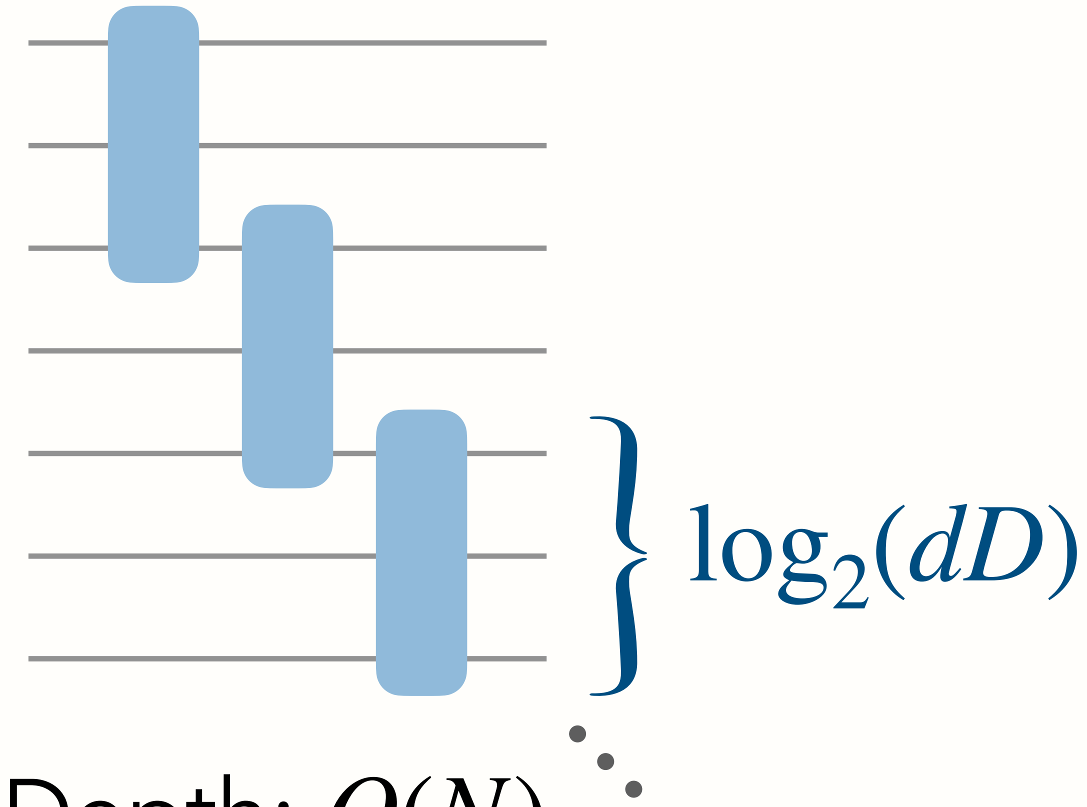




C. Schön et al, PRL (2005)

Sequential algorithm:



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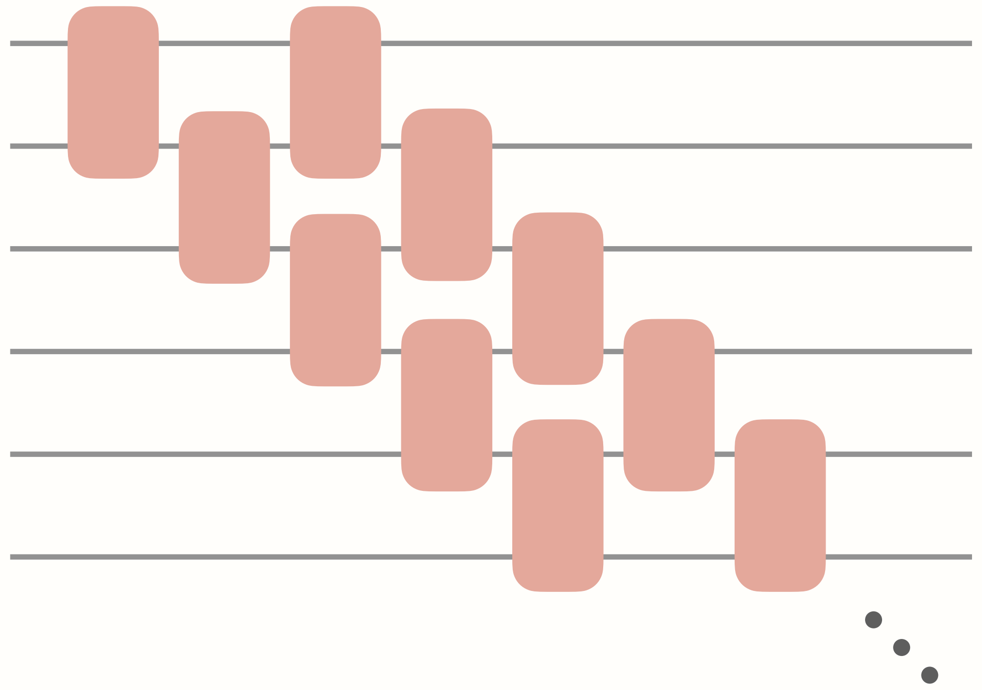
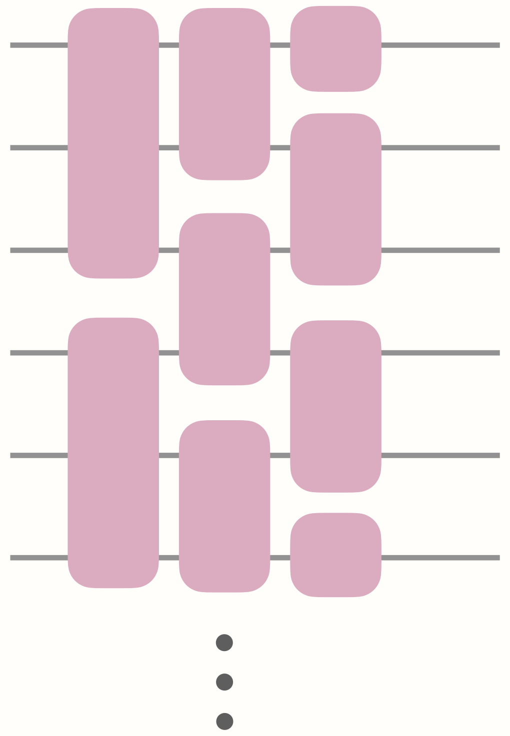
For large D:



Depth: $O(N)$

Ran et al., PRA (2020)

Short-range correlations:



Depth: $O(\log N/\epsilon)$

Malz et al., PRL (2024)

Other approaches: Rudolph et al., arXiv: 2209.00595 (2023); Jaderberg et al., arXiv: 2503.09683 (2025); Wei et al., arXiv: 2503.14645 (2025)

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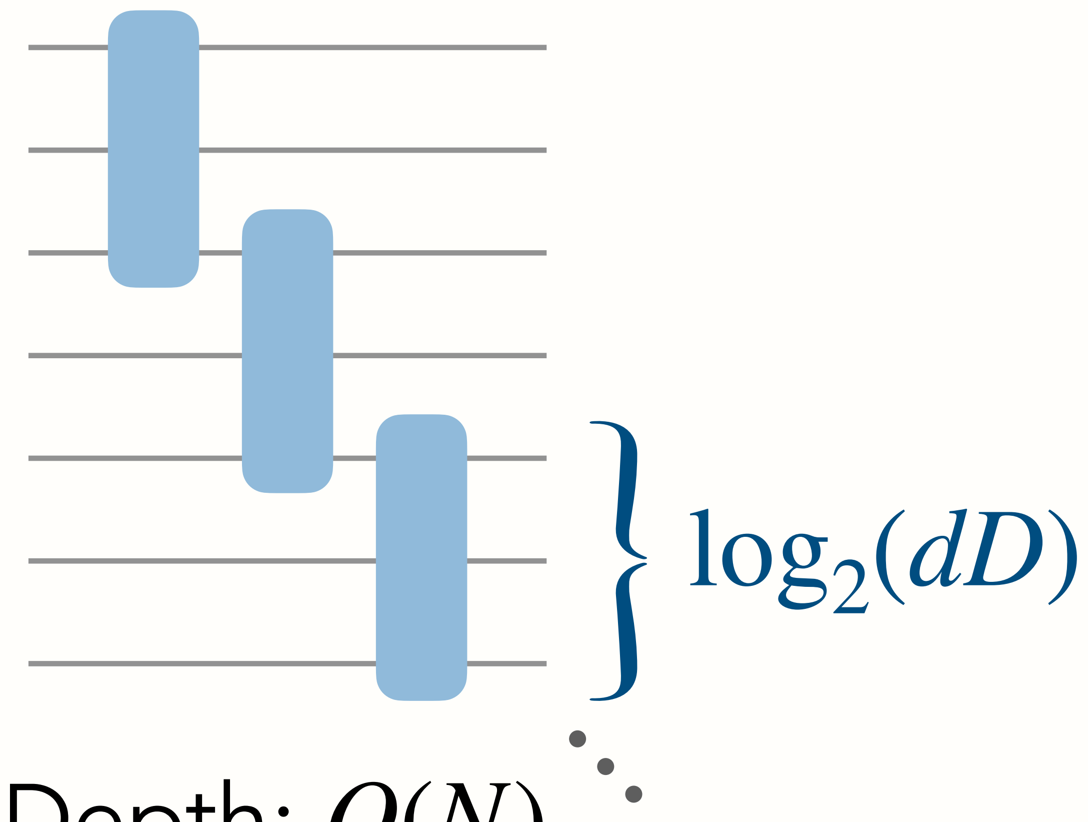

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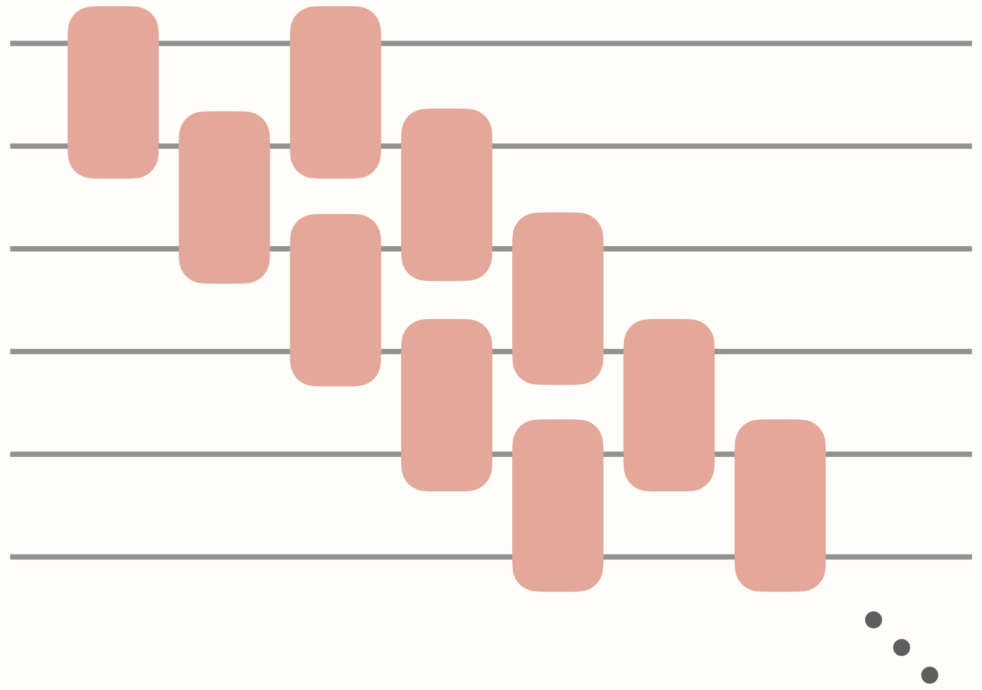

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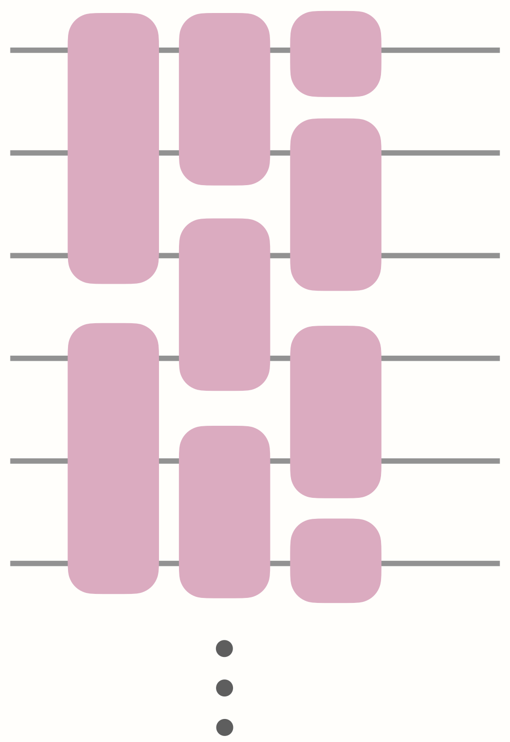

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This talk: Adaptive preparation

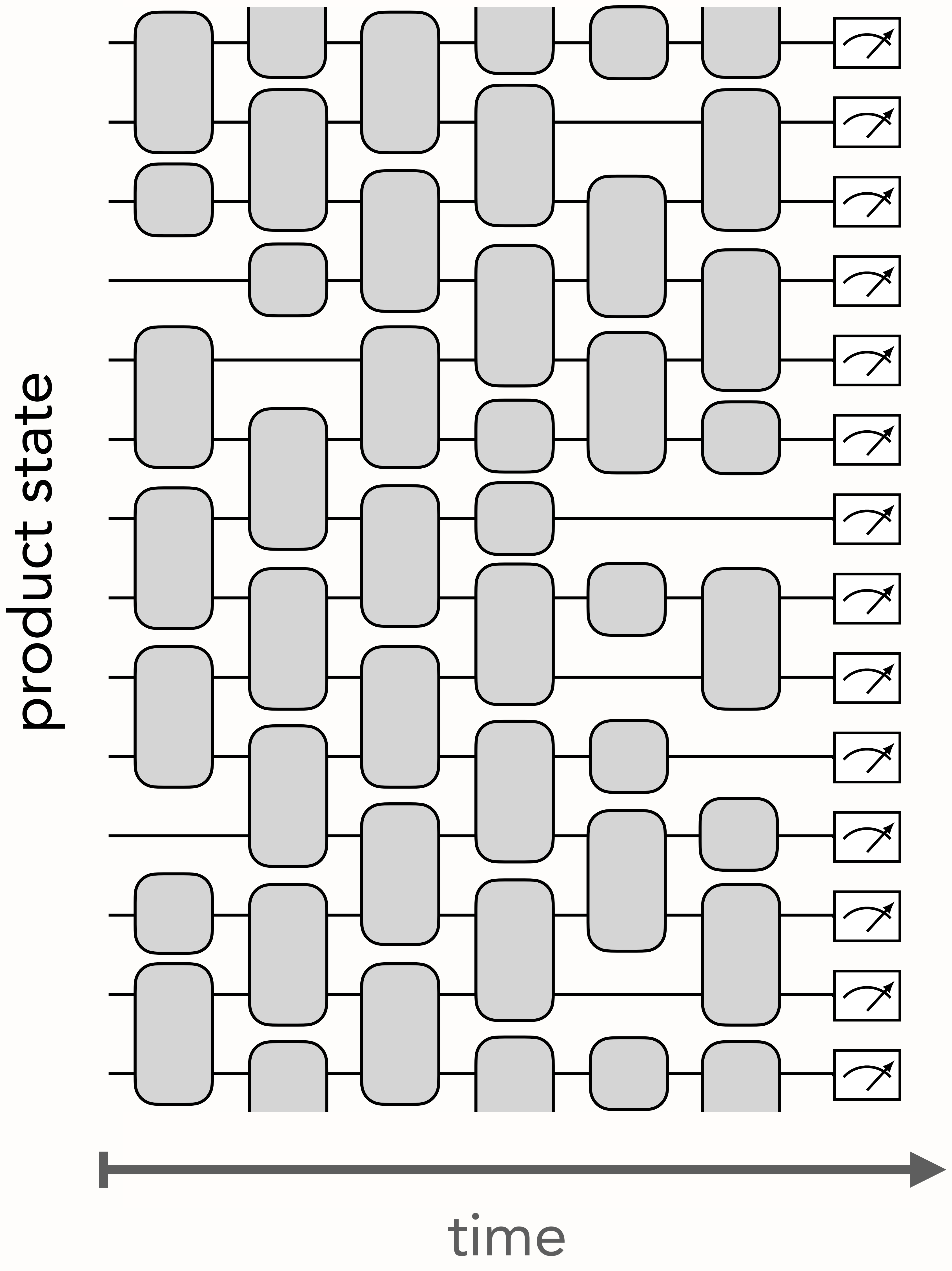
Expand our toolbox with dynamic circuits

Depth: $O(1)$ (For MPS with certain properties)

Smith, Crane, Wiebe and Girvin, PRX Quantum 4, 020315 (2023)

Smith, Khan, Clark, Wei, and Girvin, PRX Quantum 5, 030344 (2024)

Unitary quantum circuit:



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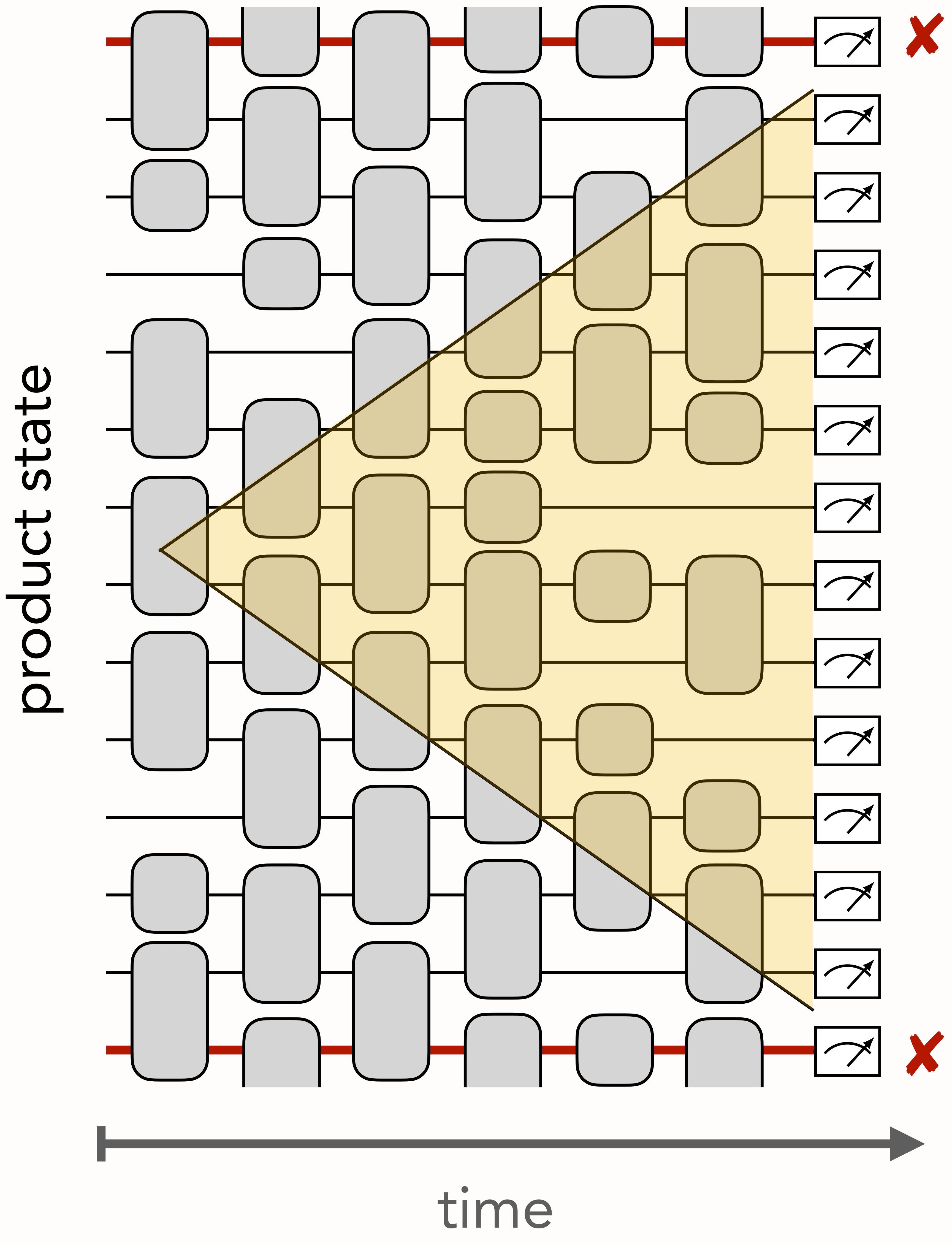

- Depth scales with system size
  - linear connectivity: $t \sim O(N)$

Bravyi et al., PRL (2006)

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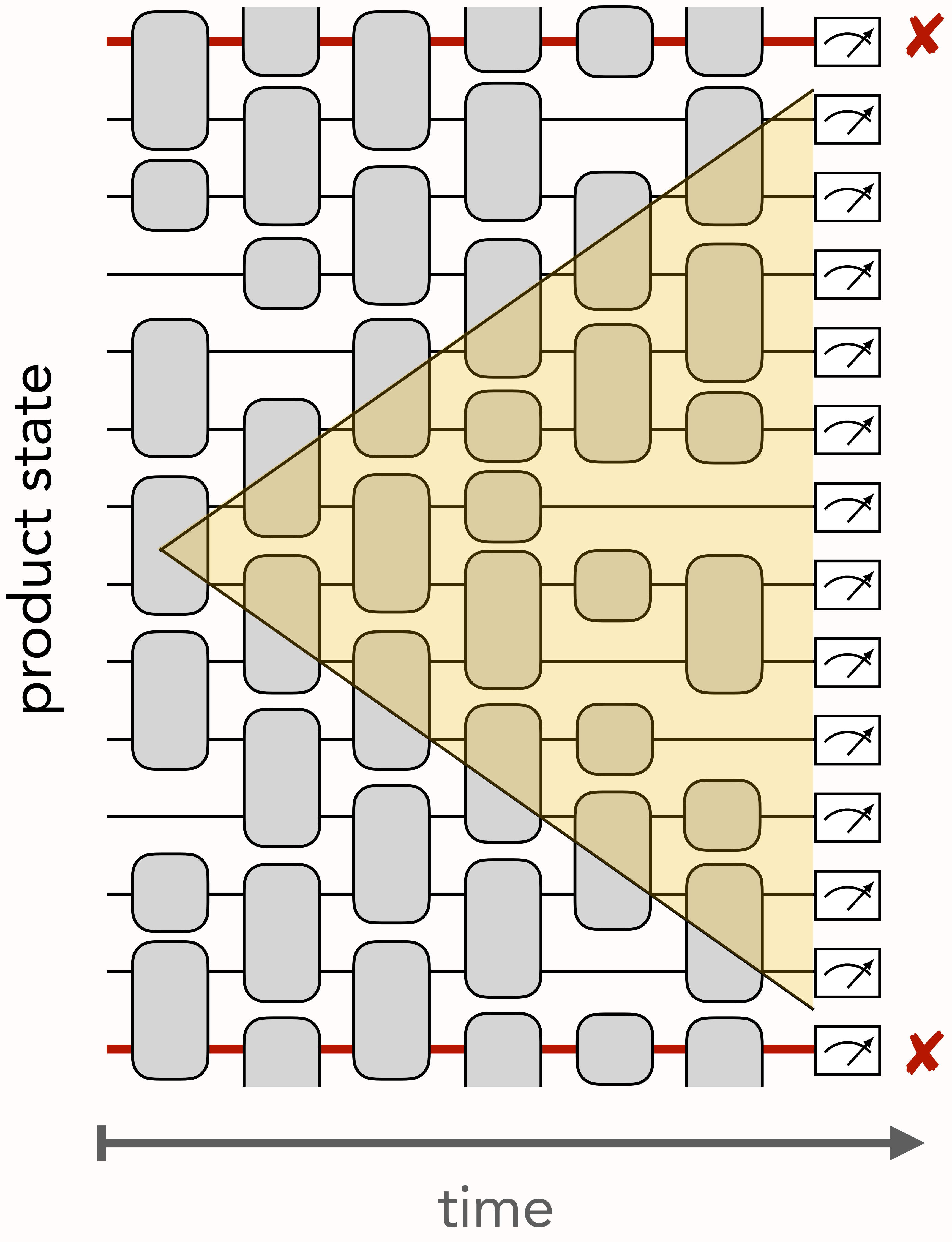

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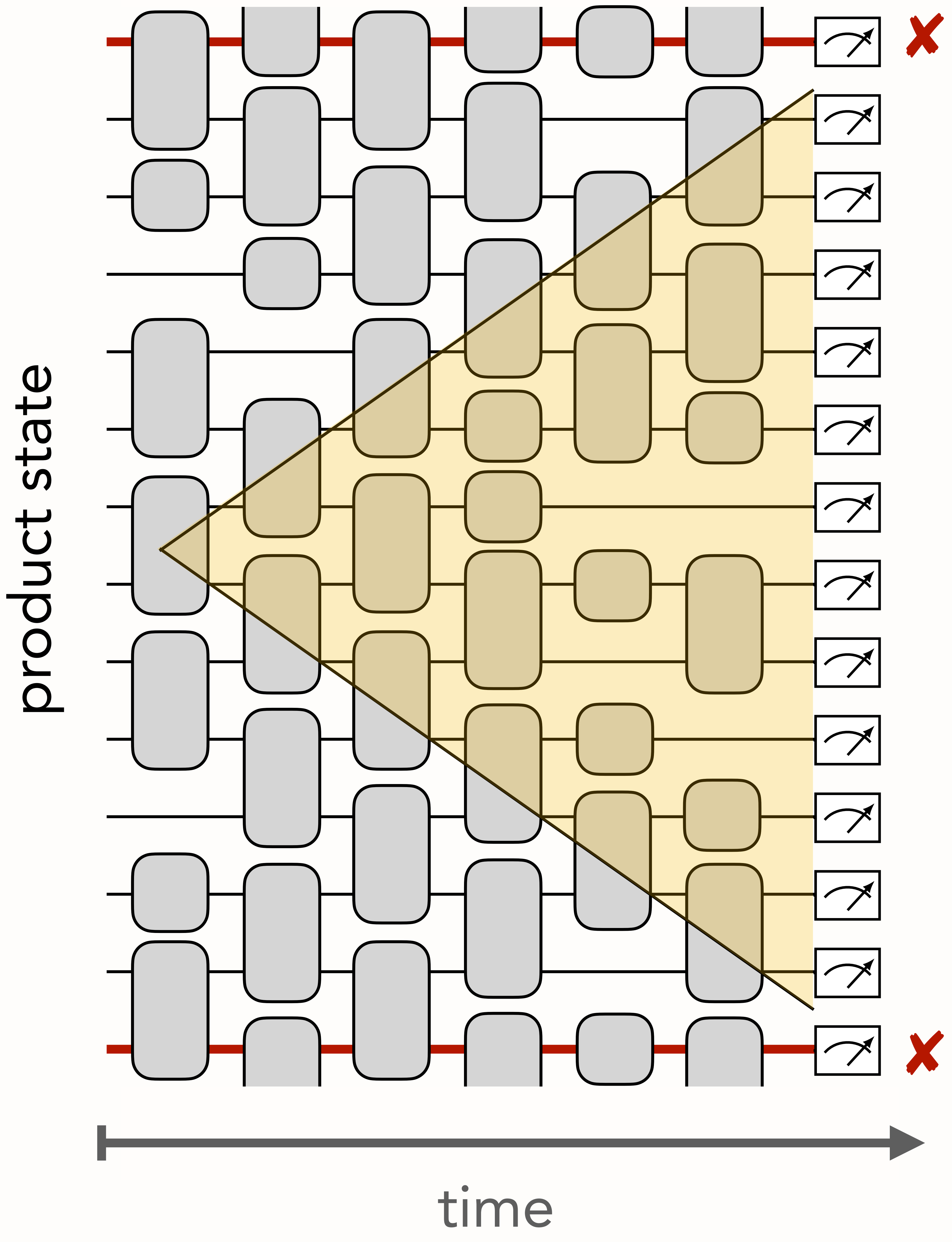

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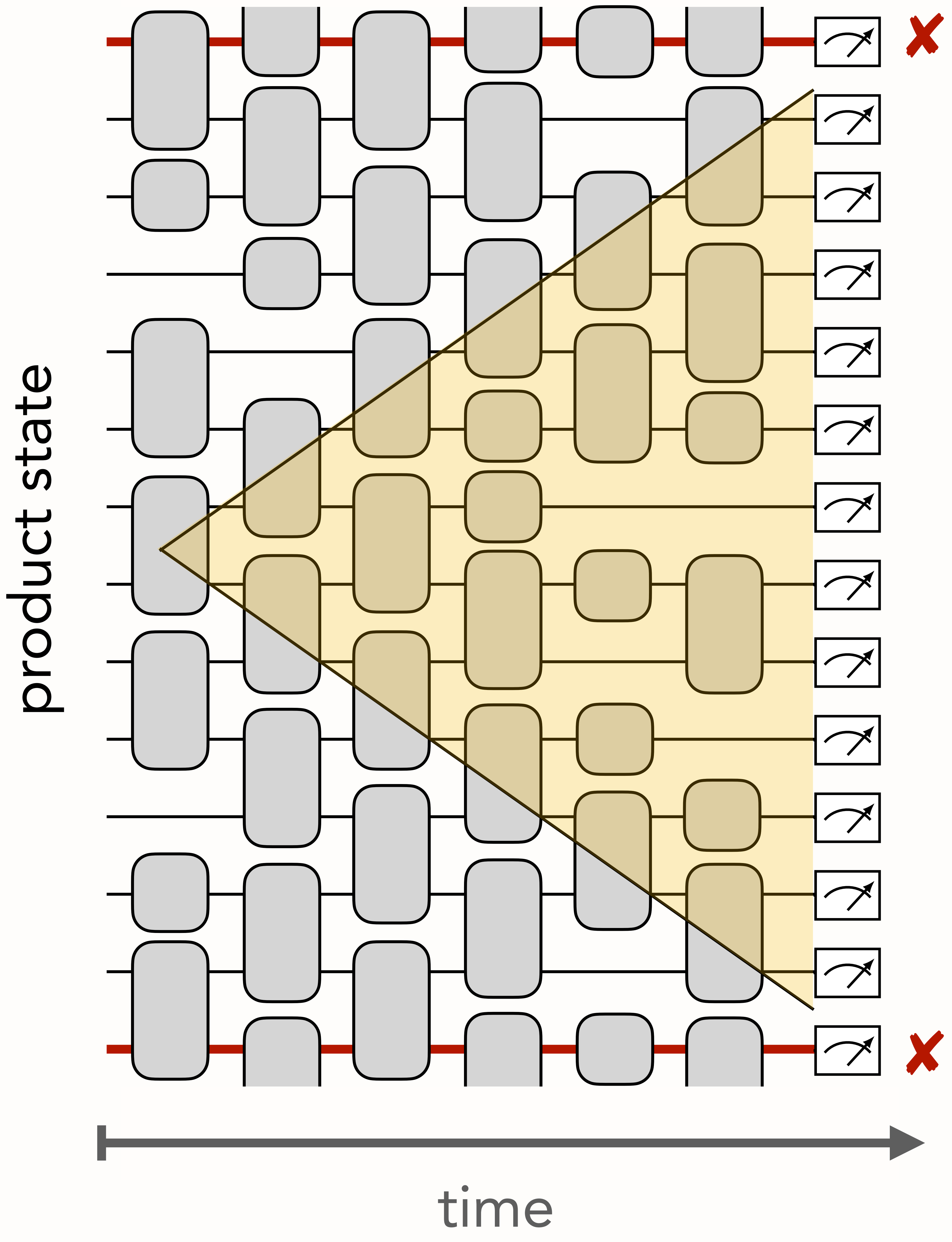

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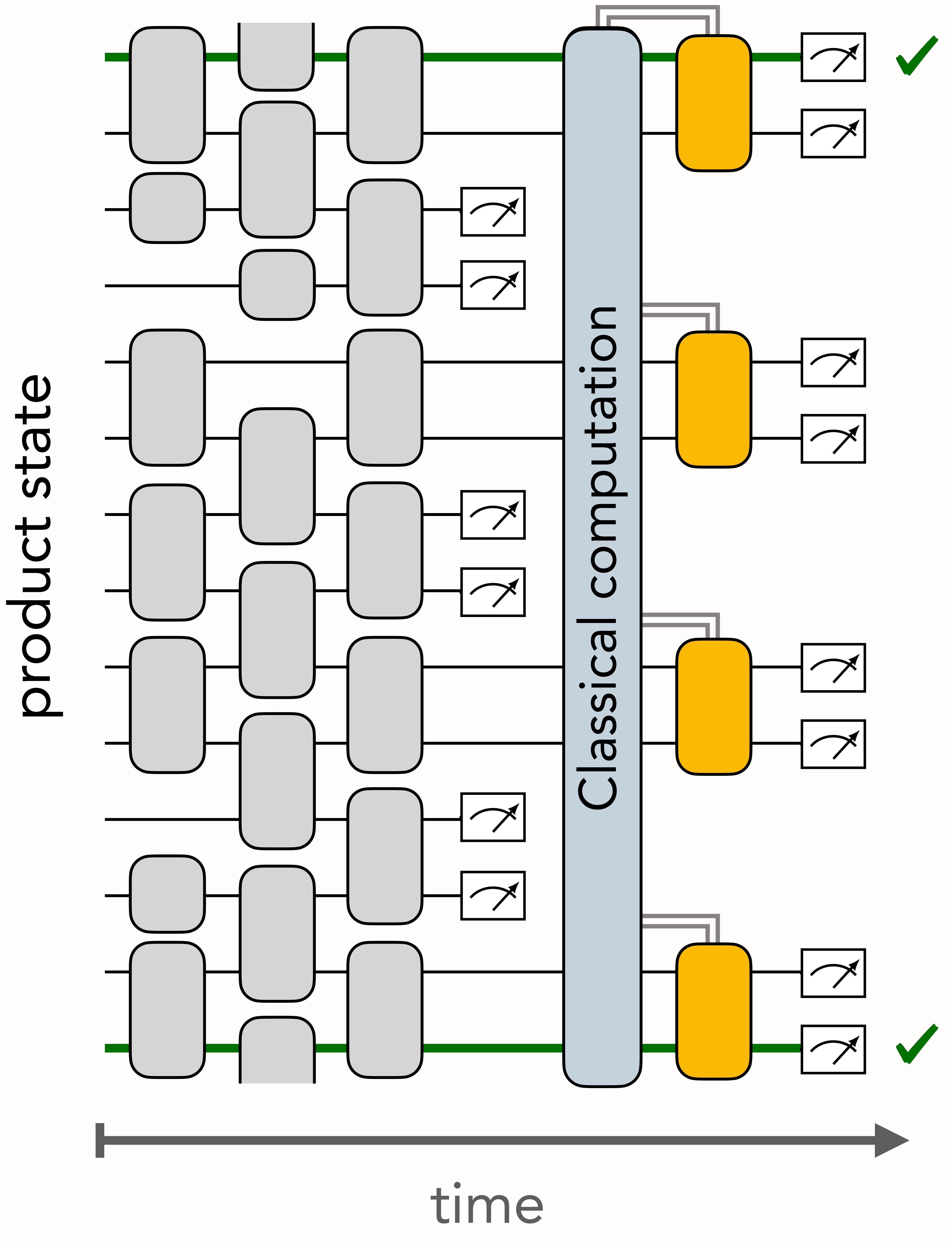
How do we approach large-scale problems (>100 qubits) with low-depth circuits?

Bravyi et al., PRL (2006)

Dynamic quantum circuit:



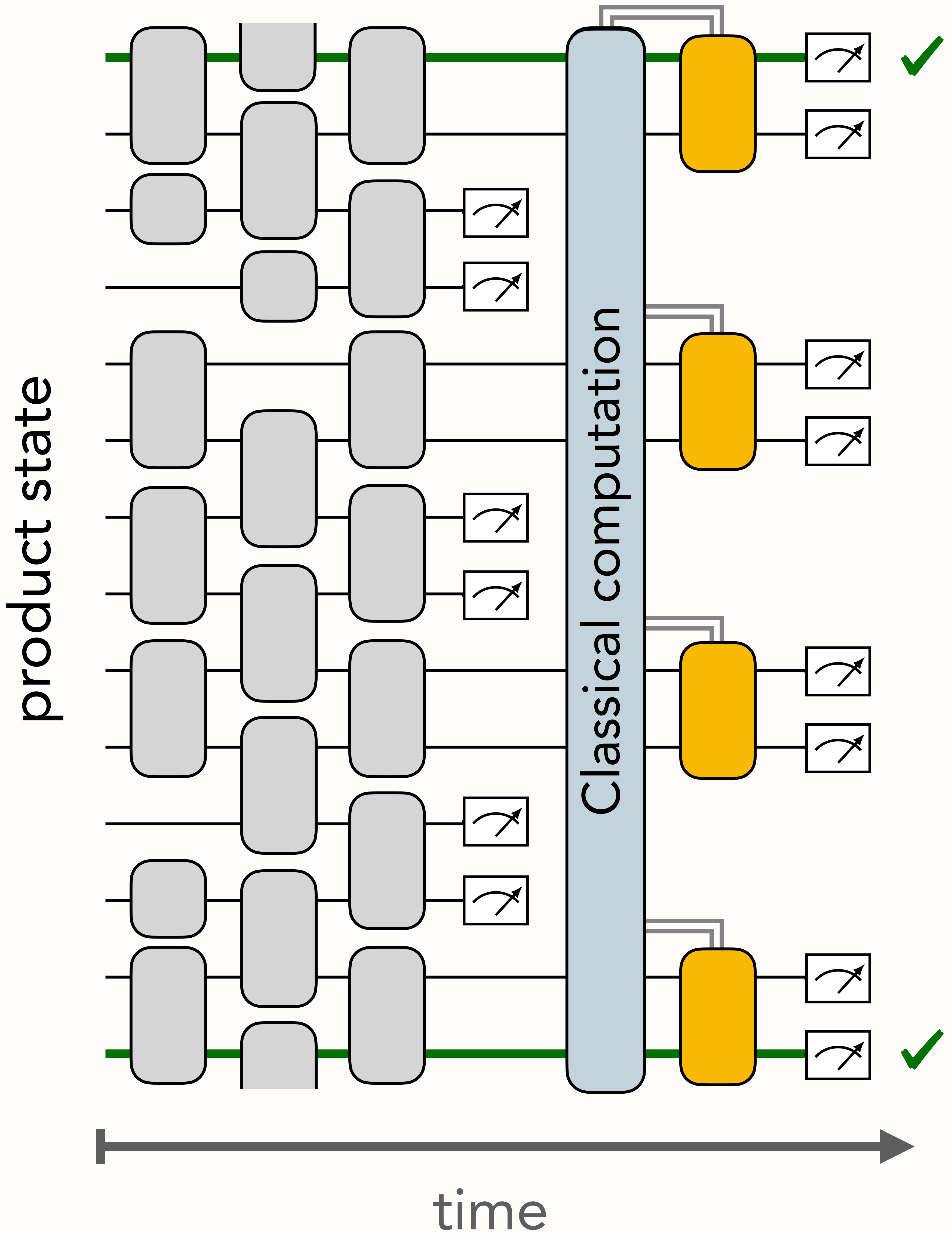
- Depth scales with system size
  - linear connectivity: $t \sim O(N)$
  - all-to-all connectivity: $t \sim O(\log(N))$
- NISQ-era: decoherence severely limits depth…

How do we approach large-scale problems (>100 qubits) with low-depth circuits?

One promising strategy:

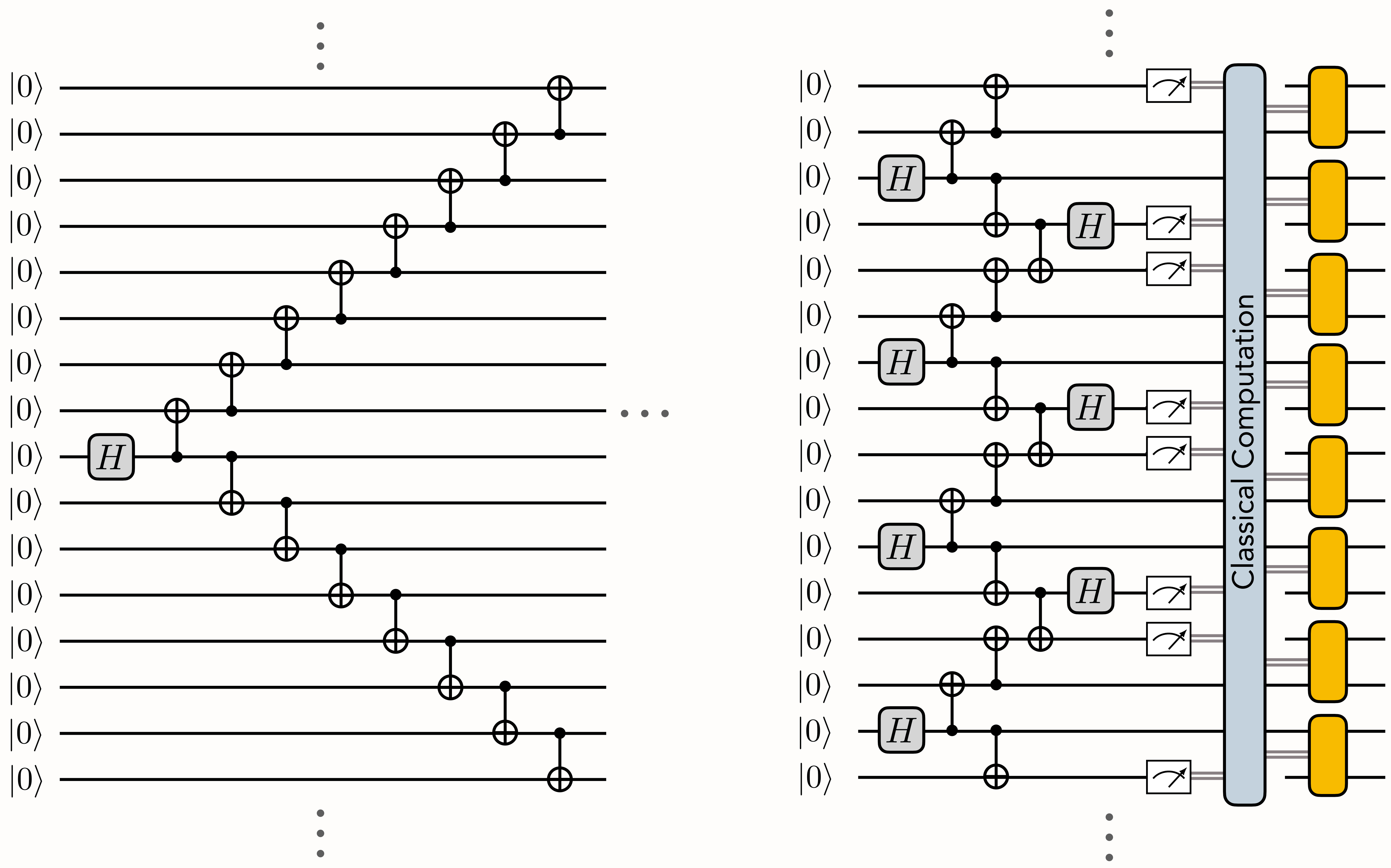
Expand our toolbox to include mid-circuit measurements + feedforward

Dynamic quantum circuit:



Example: GHZ state

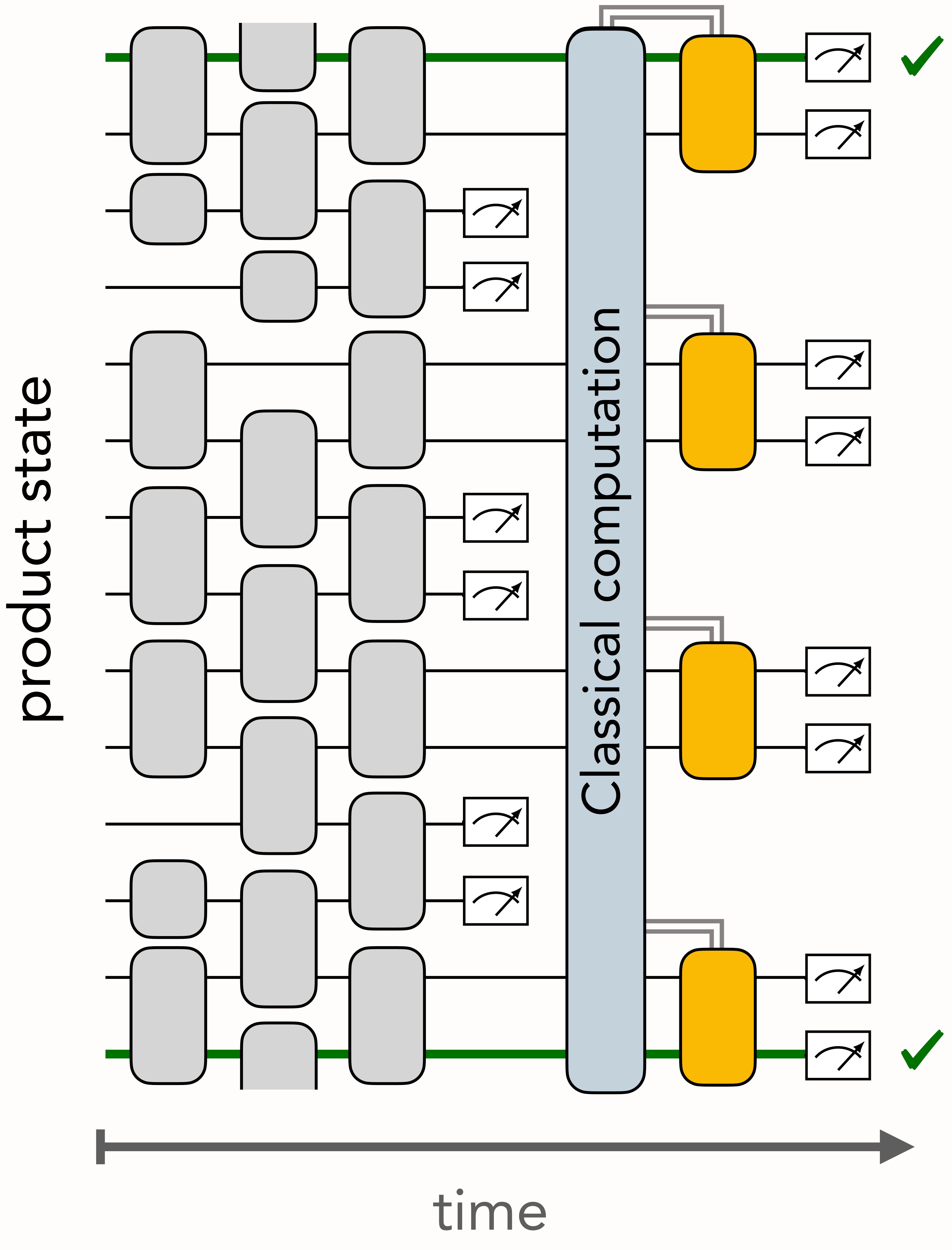
$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|000\ldots0\rangle + |111\ldots1\rangle)$$



$T = O(N)$
linear depth

$T = O(1)$
constant depth

Dynamic quantum circuit:



Many recent applications in state preparation

Theory and **Experiment**

Verresen *et al.*, **arXiv: 2112.03061** (2021)

Tantivasadakarn *et al.*, **PRX Quantum 4, 020339** (2023)

Lu *et al*, **PRX Quantum 3, 040337** (2023)

Buhrman *et al*, **arXiv: 2307.14840** (2023)

Smith *et al.*, **PRX Quantum 4, 020315** (2023)

Iqbal *et al.*, **Nat. Comm. Phys.** (2024) and **Nature 626, 505–511** (2024)

Foss-Feig *et al.*, **arXiv: 2302.03029** (2023)

Bäumer *et al.*, **PRX Quantum** (2024)

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Smith *et al.*, **PRX Quantum 5, 030344** (2024)

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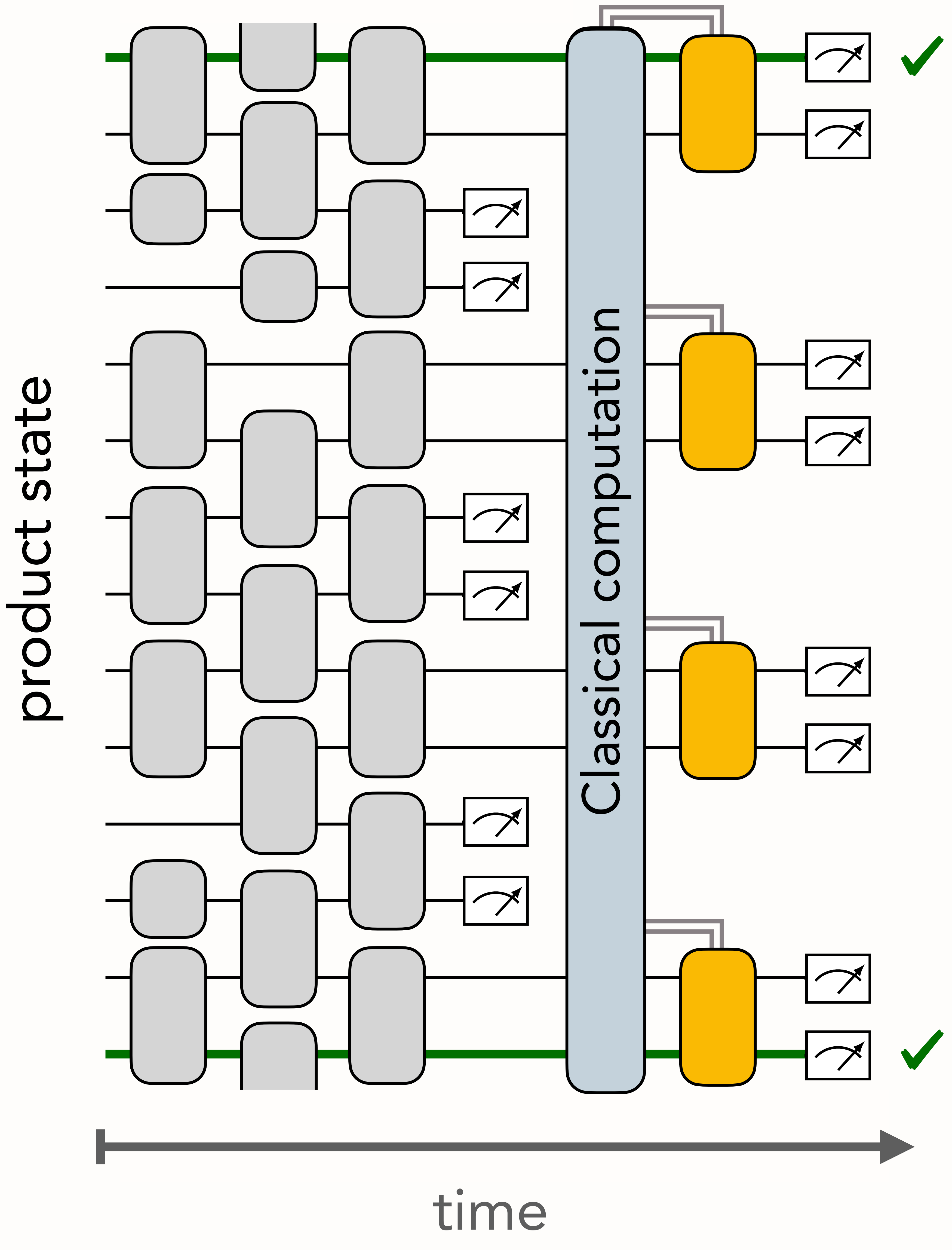

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Smith *et al.*, **PRX Quantum 4, 020315** (2023) → AKLT state

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Chen *et al.*, **arXiv: 2309.02863** (2023)

Smith *et al.*, **PRX Quantum 5, 030344** (2024) → Generalization to other MPS

Piroli *et al.*, **arXiv: 2403.07604** (2024)

Constant-depth preparation of matrix product states

Three ingredients:

Constant-depth preparation of matrix product states

Three ingredients:

1. Unitarily prepare small MPS

Constant-depth preparation of matrix product states

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2. Mid-circuit "fusion" measurements

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[1] Pollman et al., PRB (2012)

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- Symmetry-protected topological order [1]
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Example: Affleck-Kennedy-Lieb-Tasaki (AKLT) state

- Symmetry-protected topological order [1]
- Resource state for measurement-based quantum computation [2-3]
- Non-zero correlation length: faithful unitary preparation requires depth $T = \Omega(\log N/\epsilon)$ [4]

[1] Pollman et al., PRB (2012) [2] Verstraete et al., PRA (2004); [3] Brennen et al., PRL (2008); [4] Malz et al., PRL (2024)

Constant-depth preparation of matrix product states

Three ingredients:

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3. Feedforward corrections

Example: Affleck-Kennedy-Lieb-Tasaki (AKLT) state

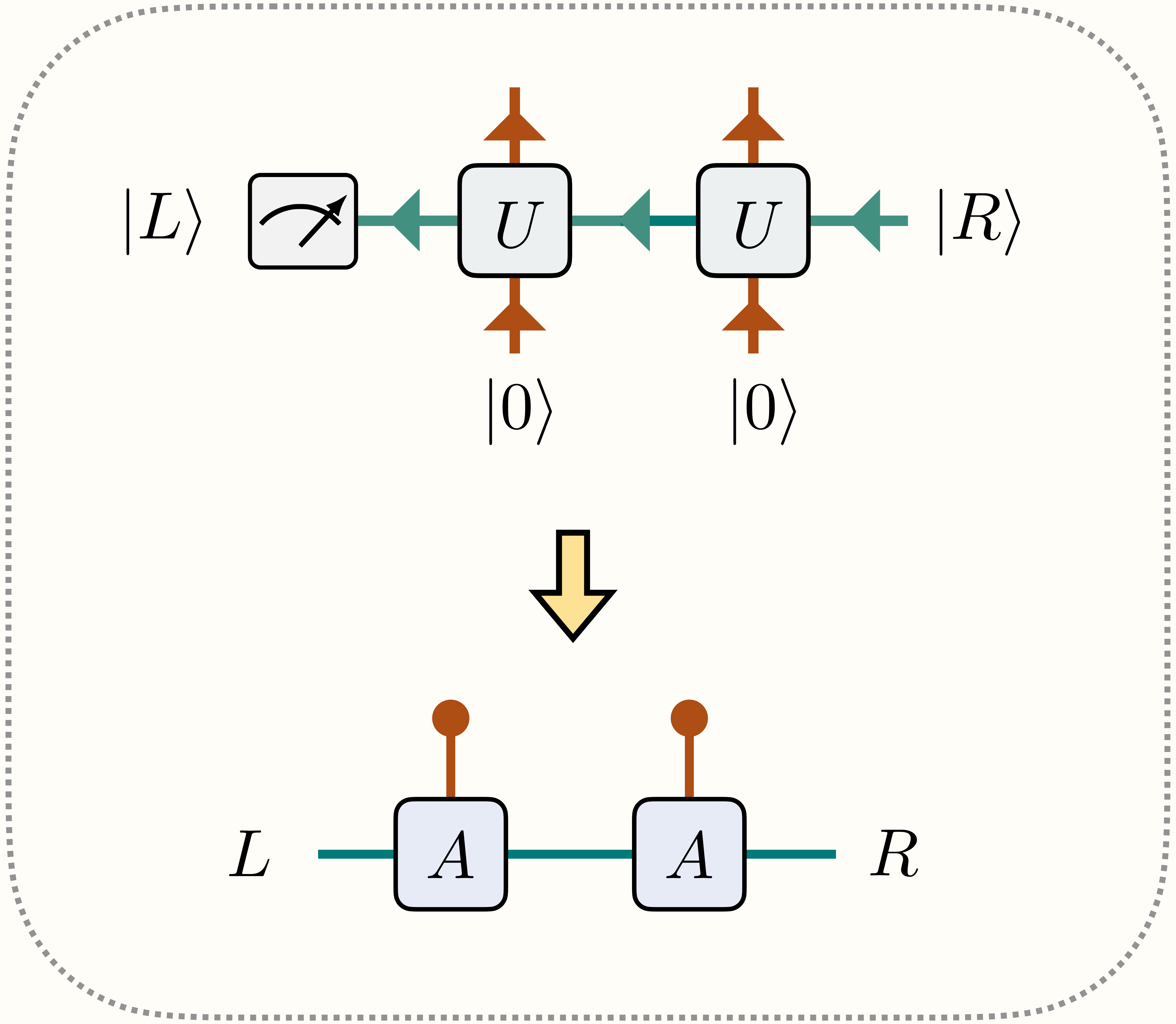
$$A^+ = \sqrt{\frac{2}{3}}\sigma^+ \qquad A^- = -\sqrt{\frac{2}{3}}\sigma^- \qquad A^0 = -\sqrt{\frac{1}{3}}\sigma^z$$

$d = 3$ (Spin-1); $D = 2$

[1] Pollman et al., PRB (2012) [2] Verstraete et al., PRA (2004); [3] Brennen et al., PRL (2008)

Ingredient #1: Unitarily Prepare Small MPS

Recall:



Smith, Crane, Wiebe and Girvin, PRX Quantum 4, 020315 (2023)

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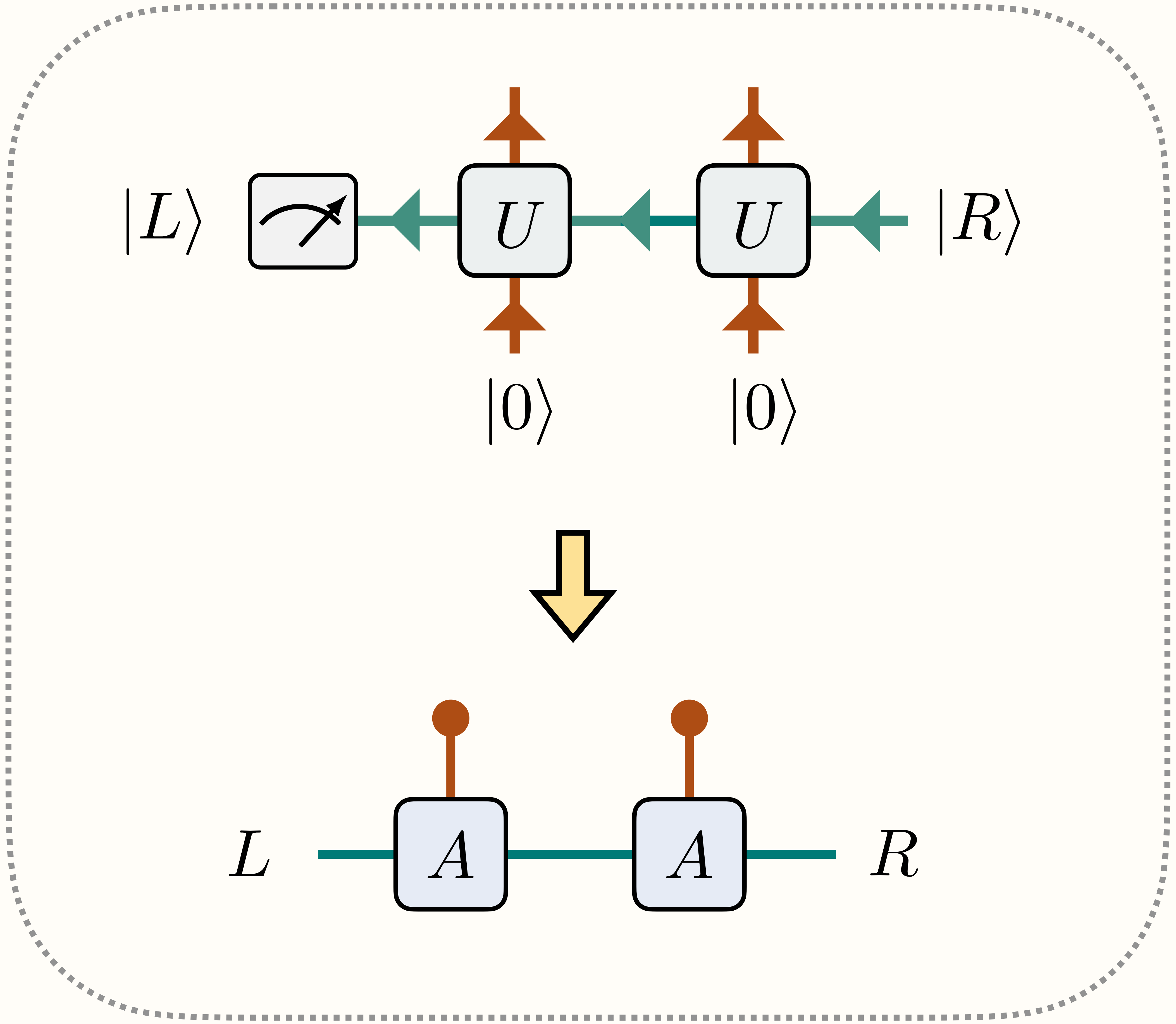
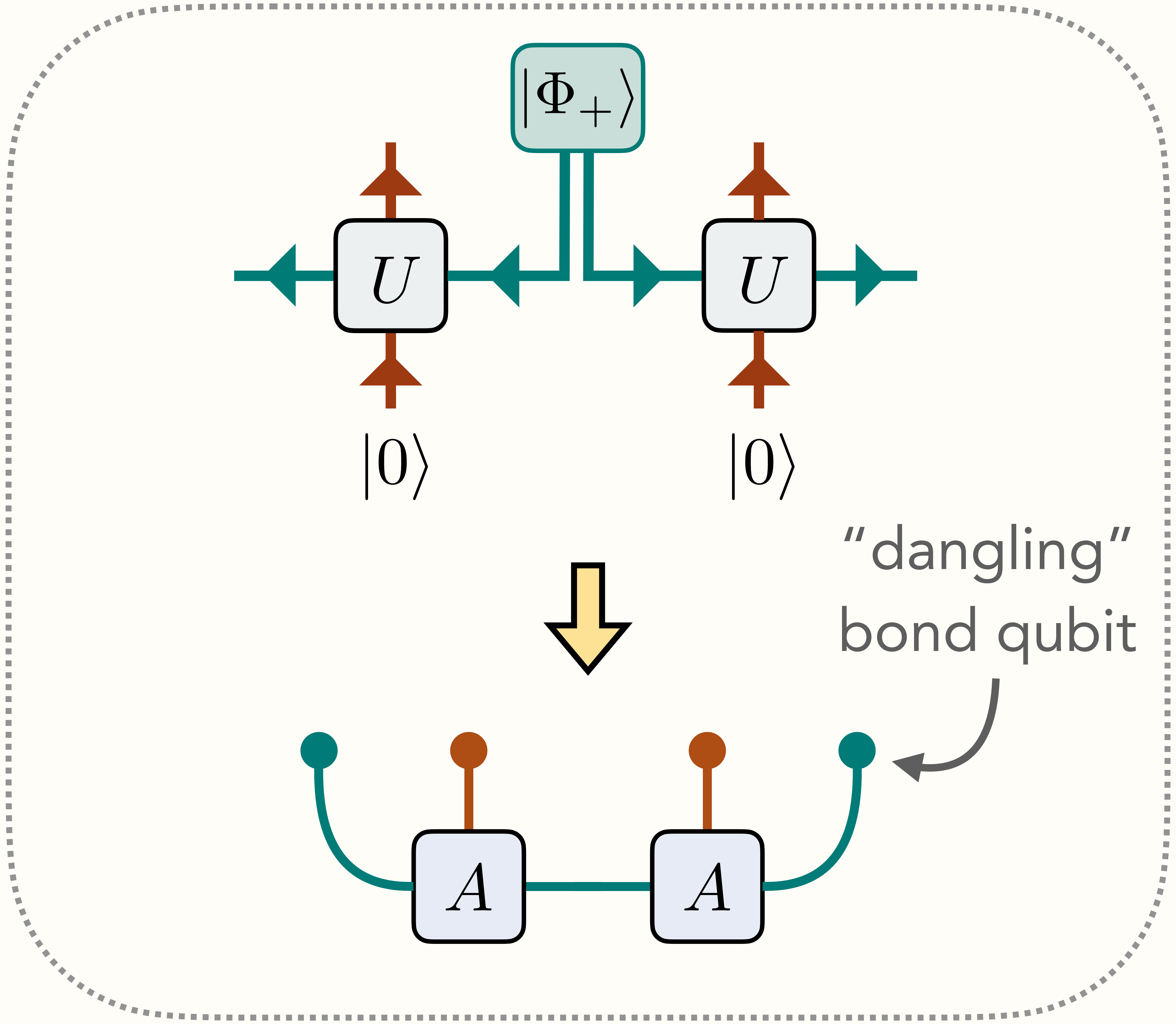


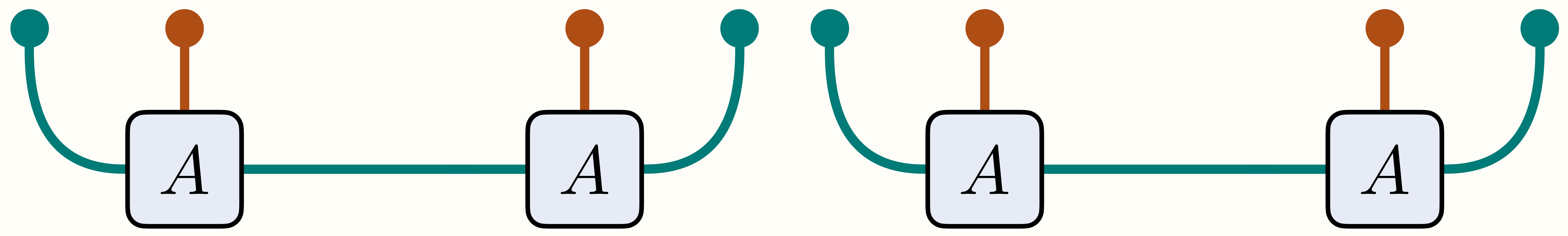
Instead, we use:



$|\Phi_+\rangle = [|00\rangle + |11\rangle]/\sqrt{2}$

Smith, Crane, Wiebe and Girvin, PRX Quantum 4, 020315 (2023)

Ingredient #2: MPS can be "fused" with measurements



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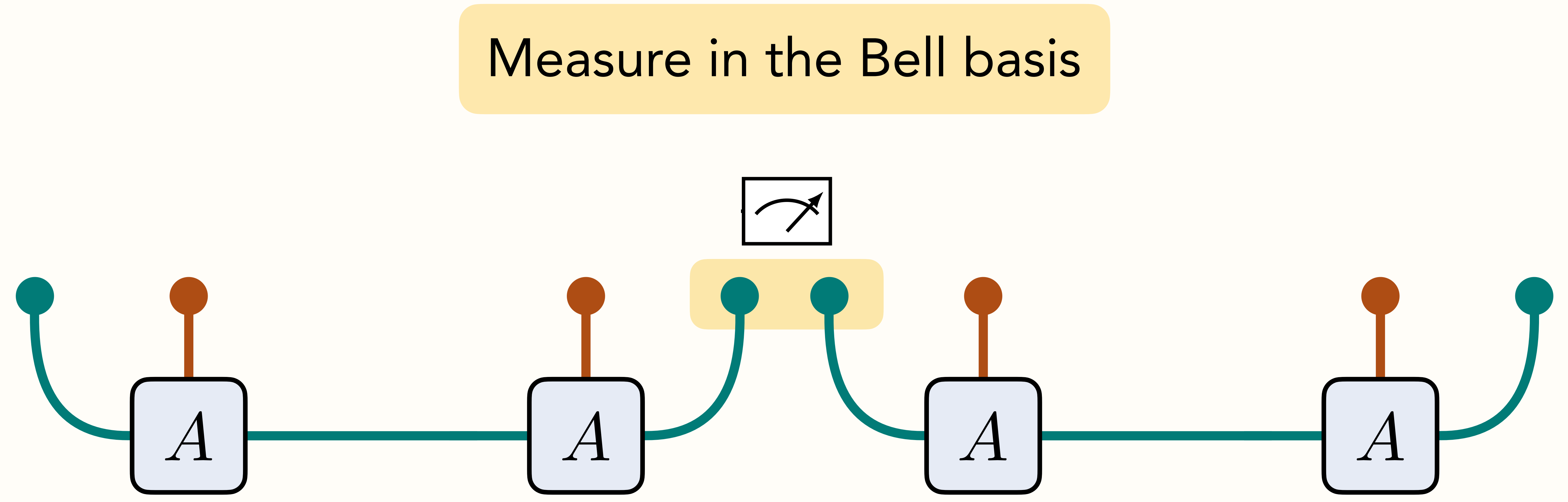

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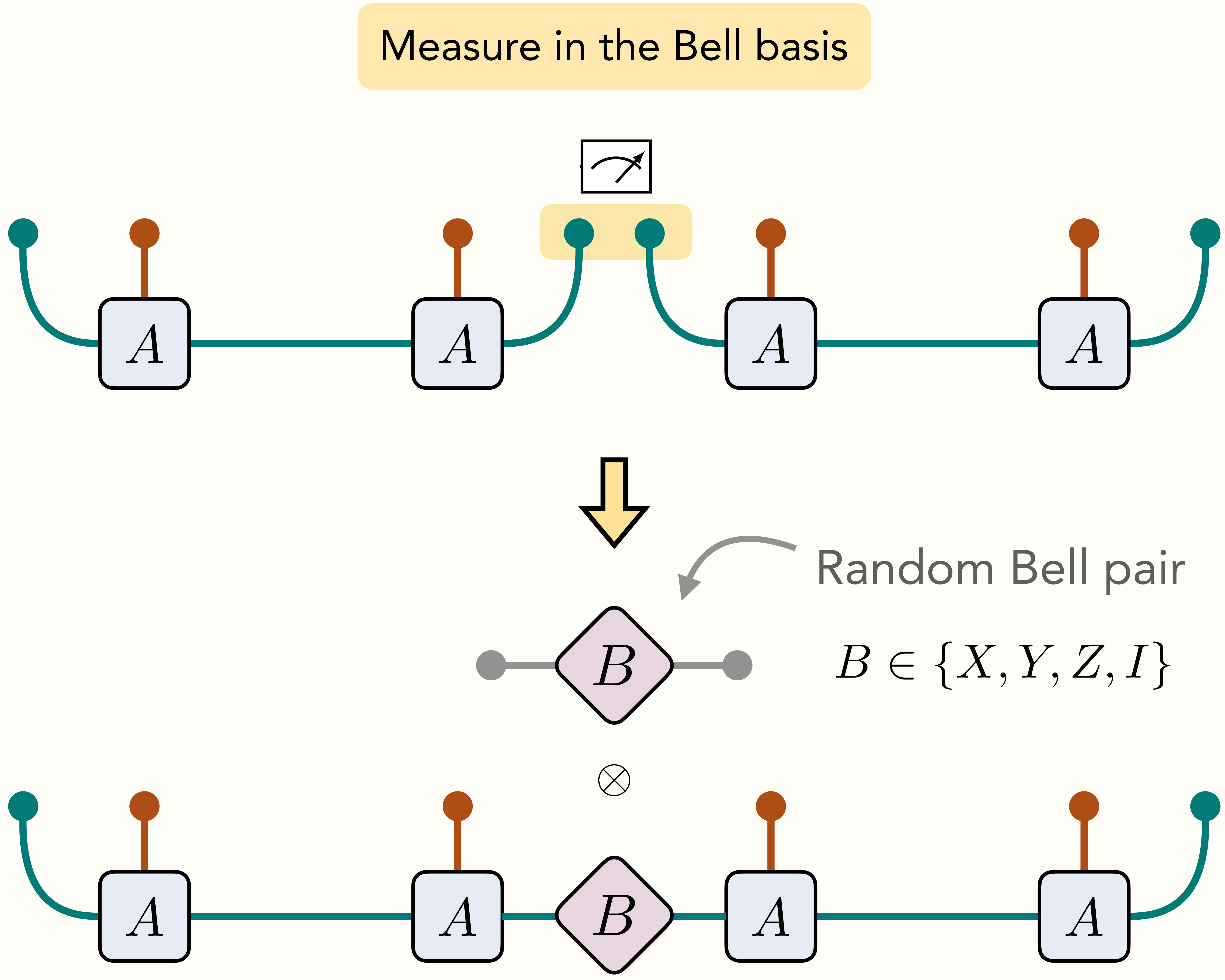
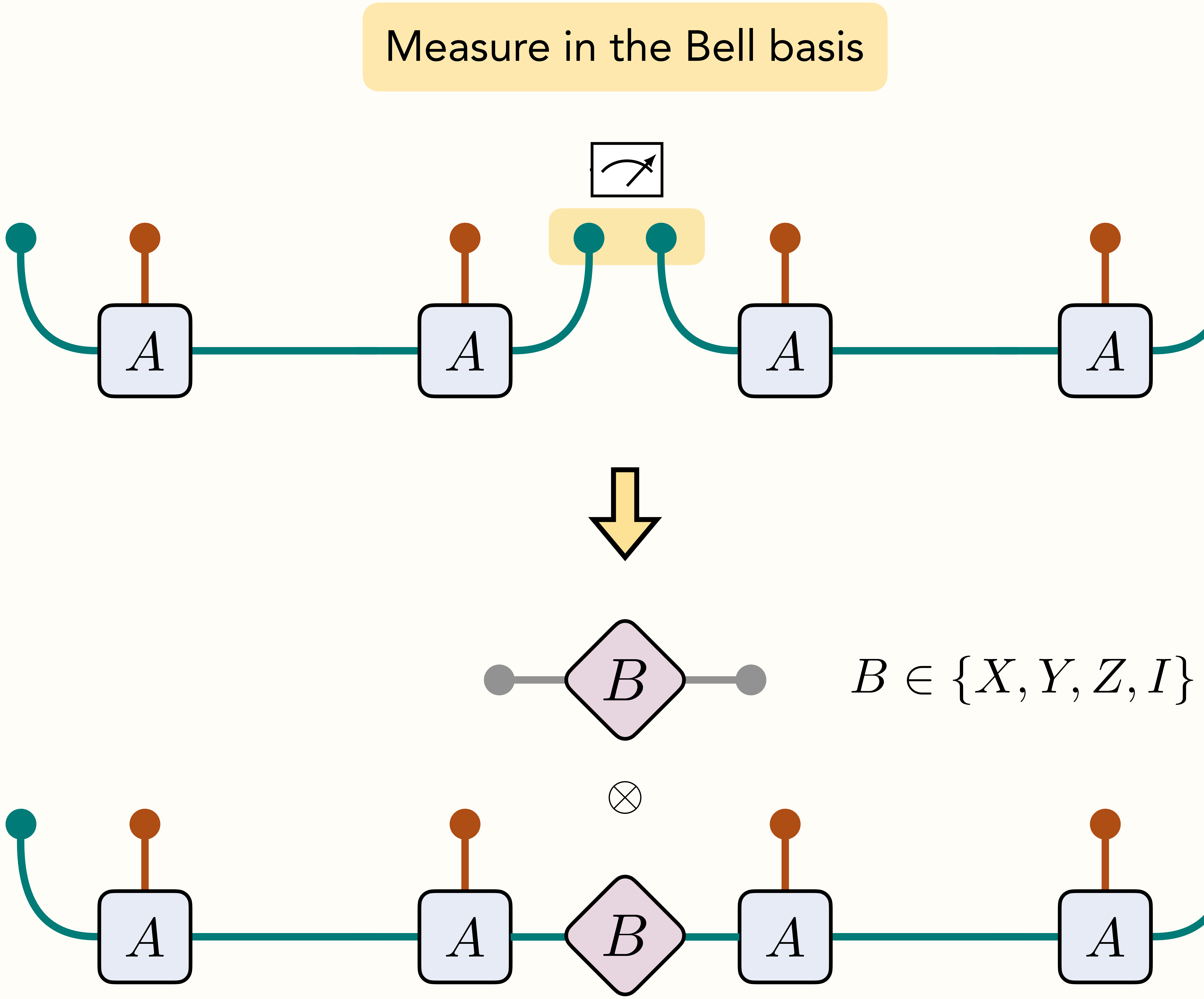
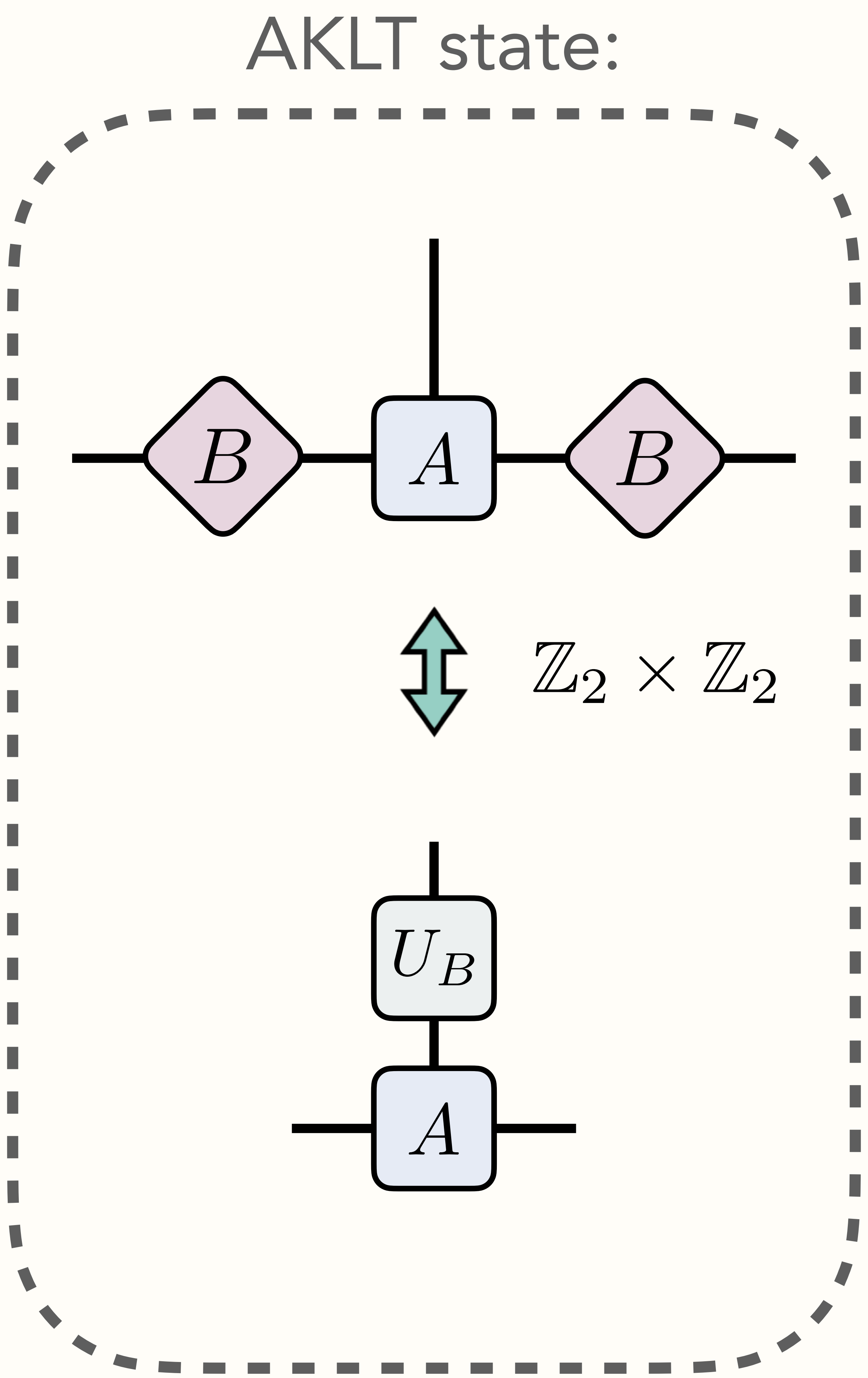


Smith, Crane, Wiebe and Girvin, PRX Quantum 4, 020315 (2023)

Ingredient #3: Defects can be corrected by leveraging symmetry





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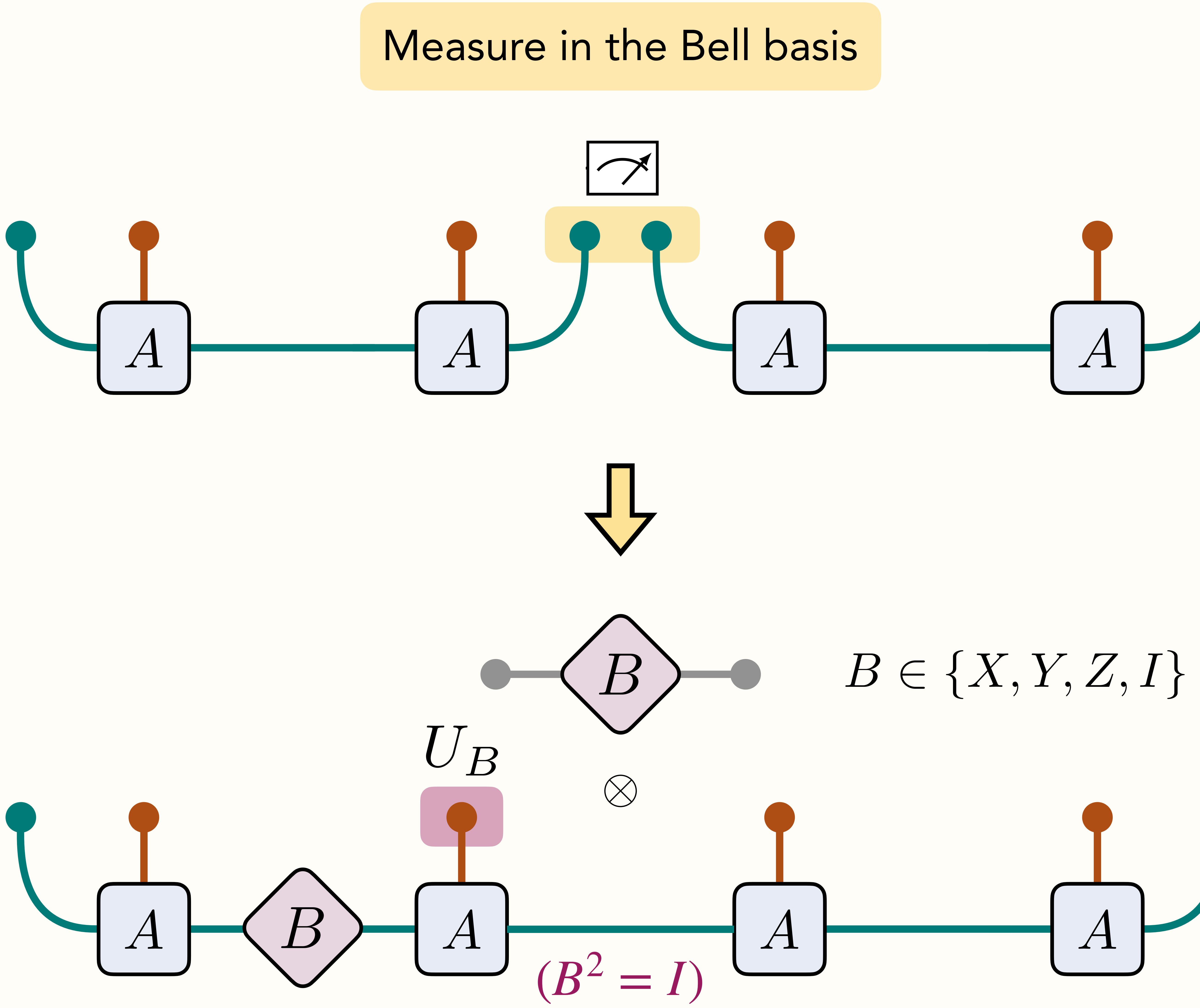

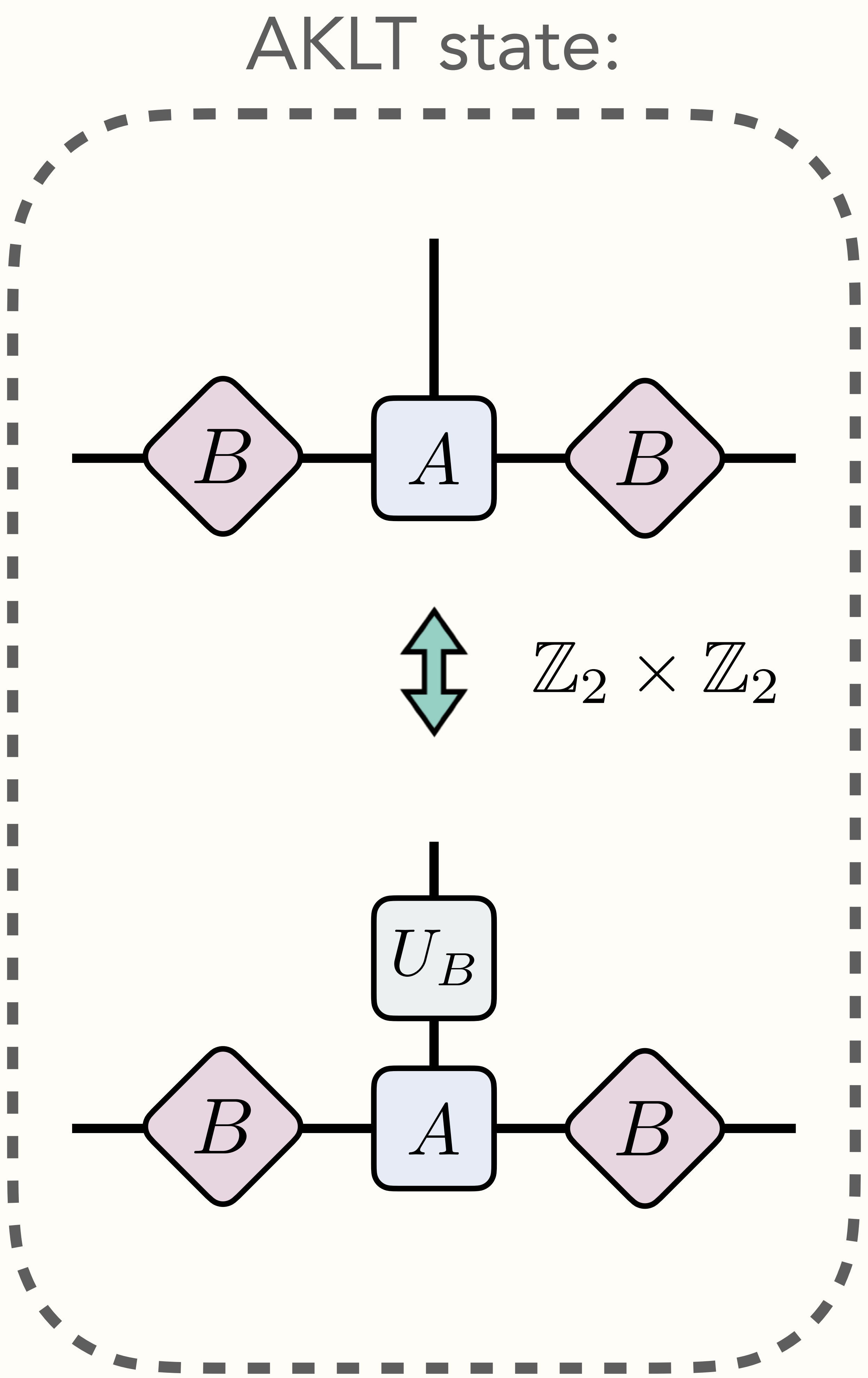

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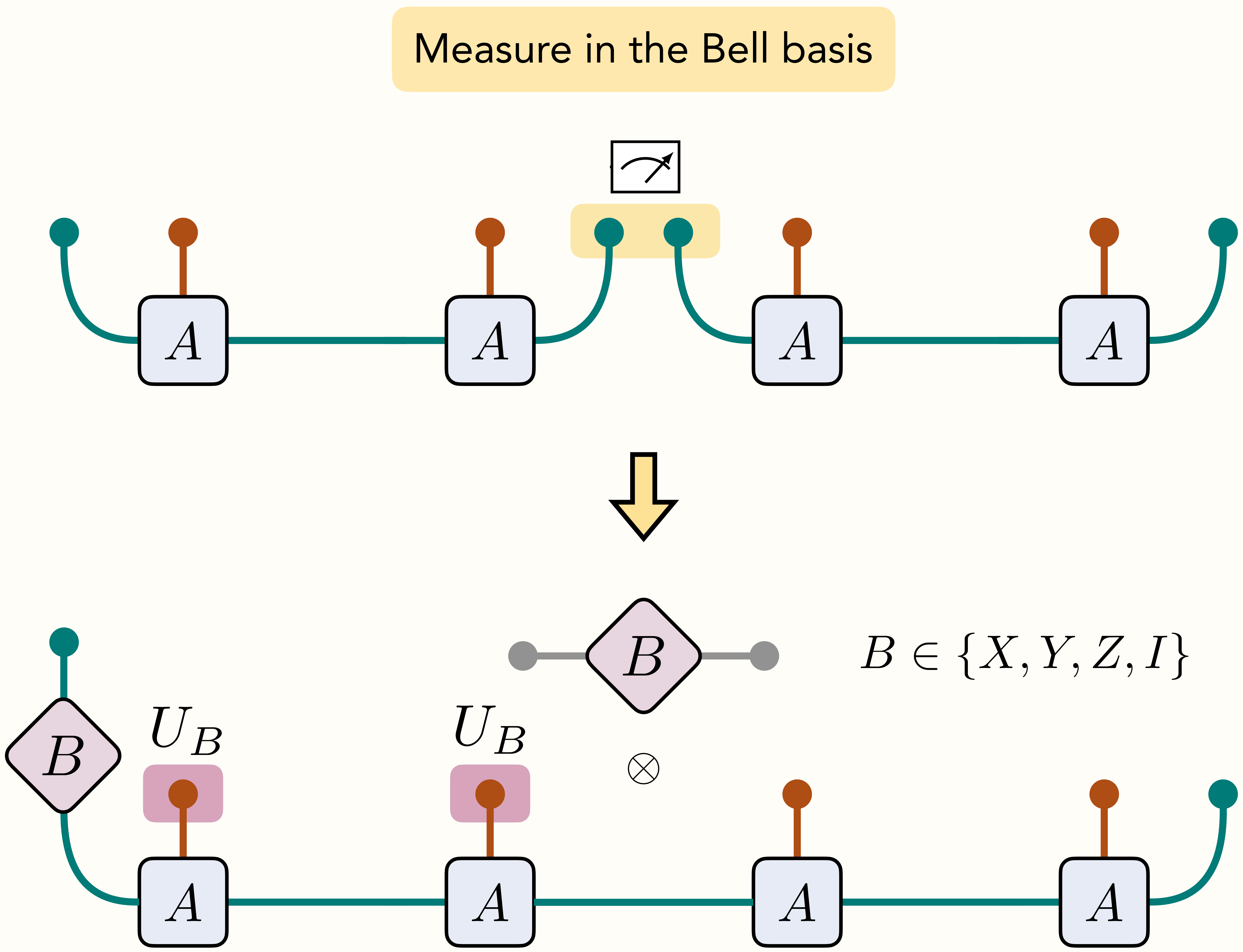

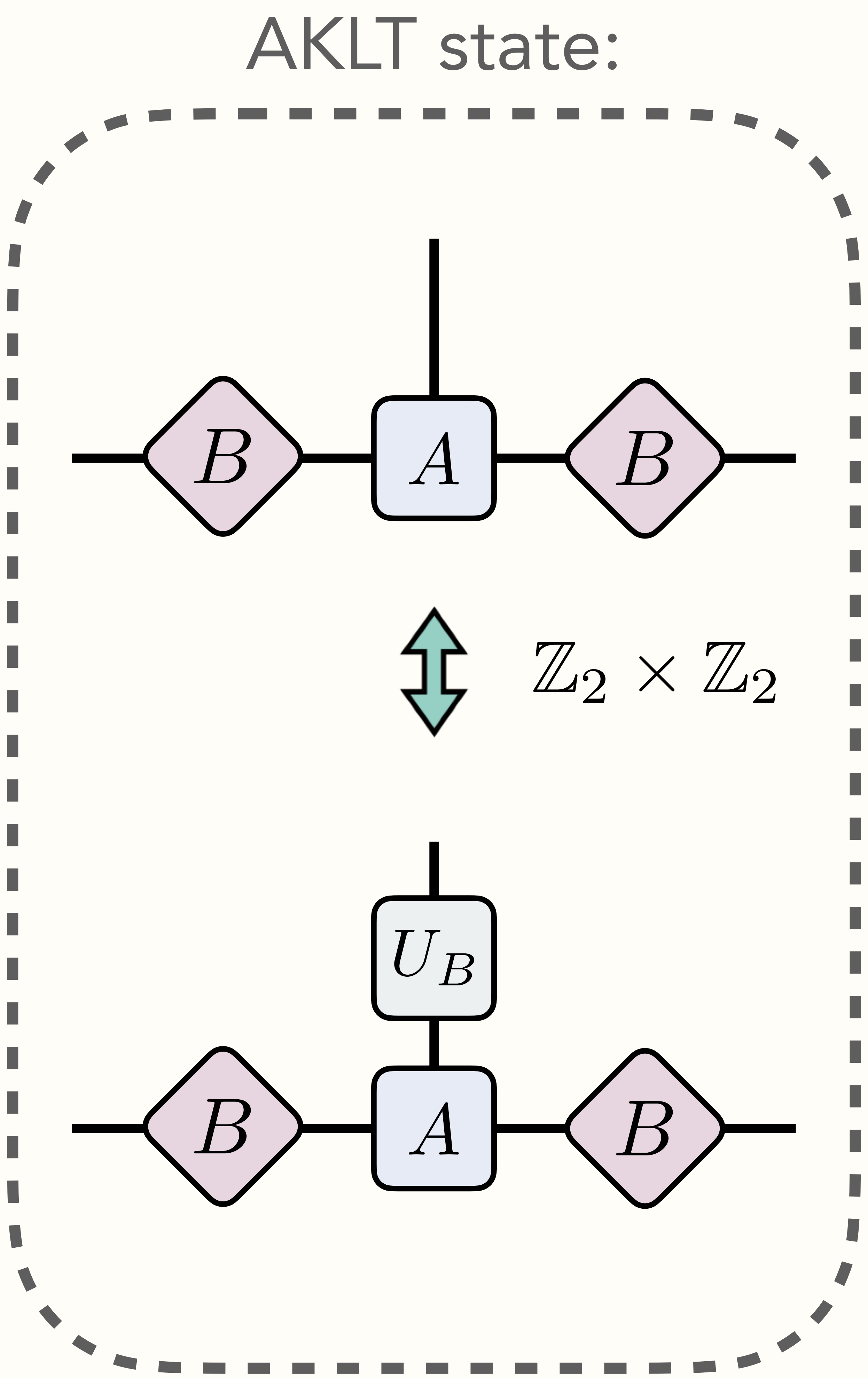

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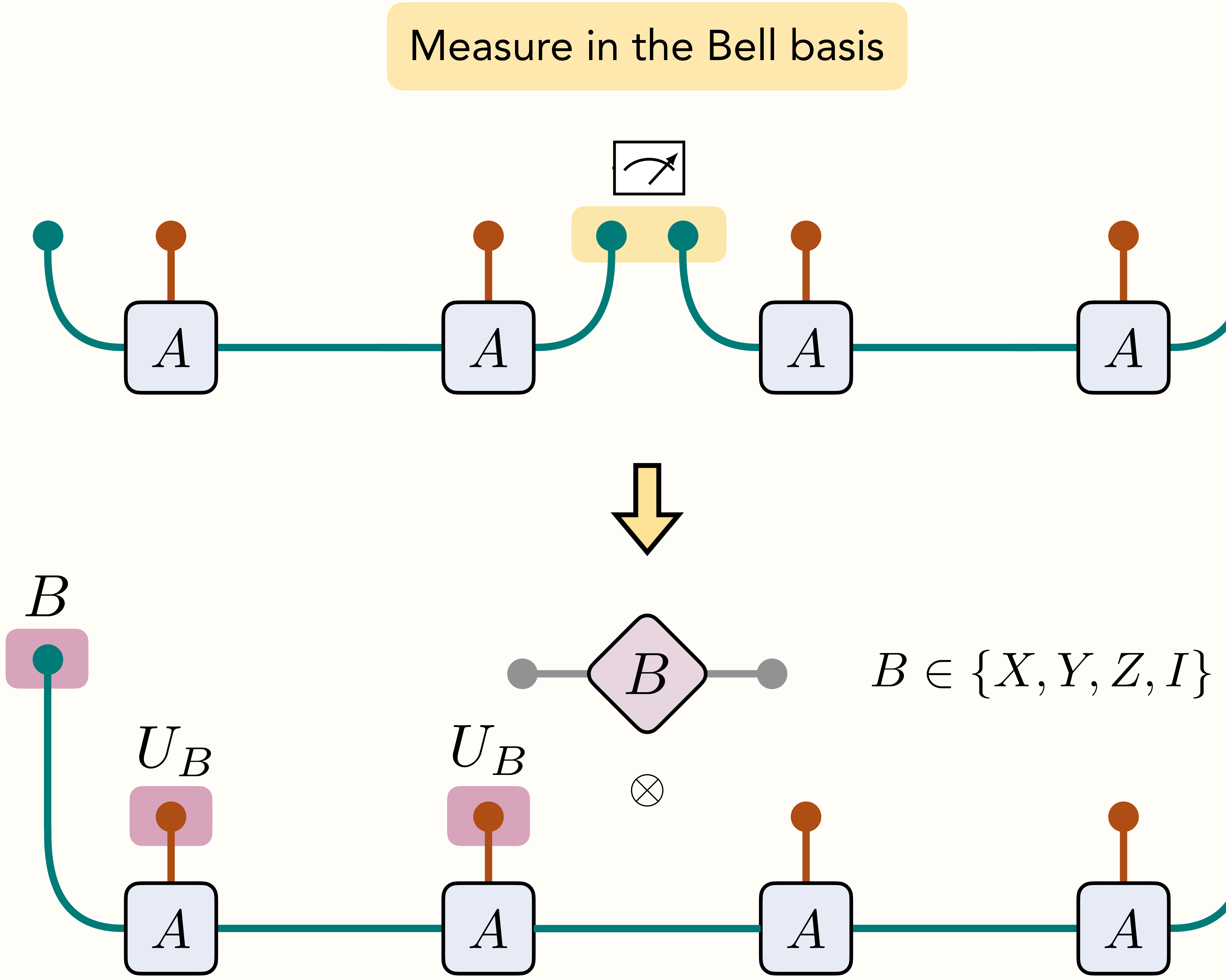

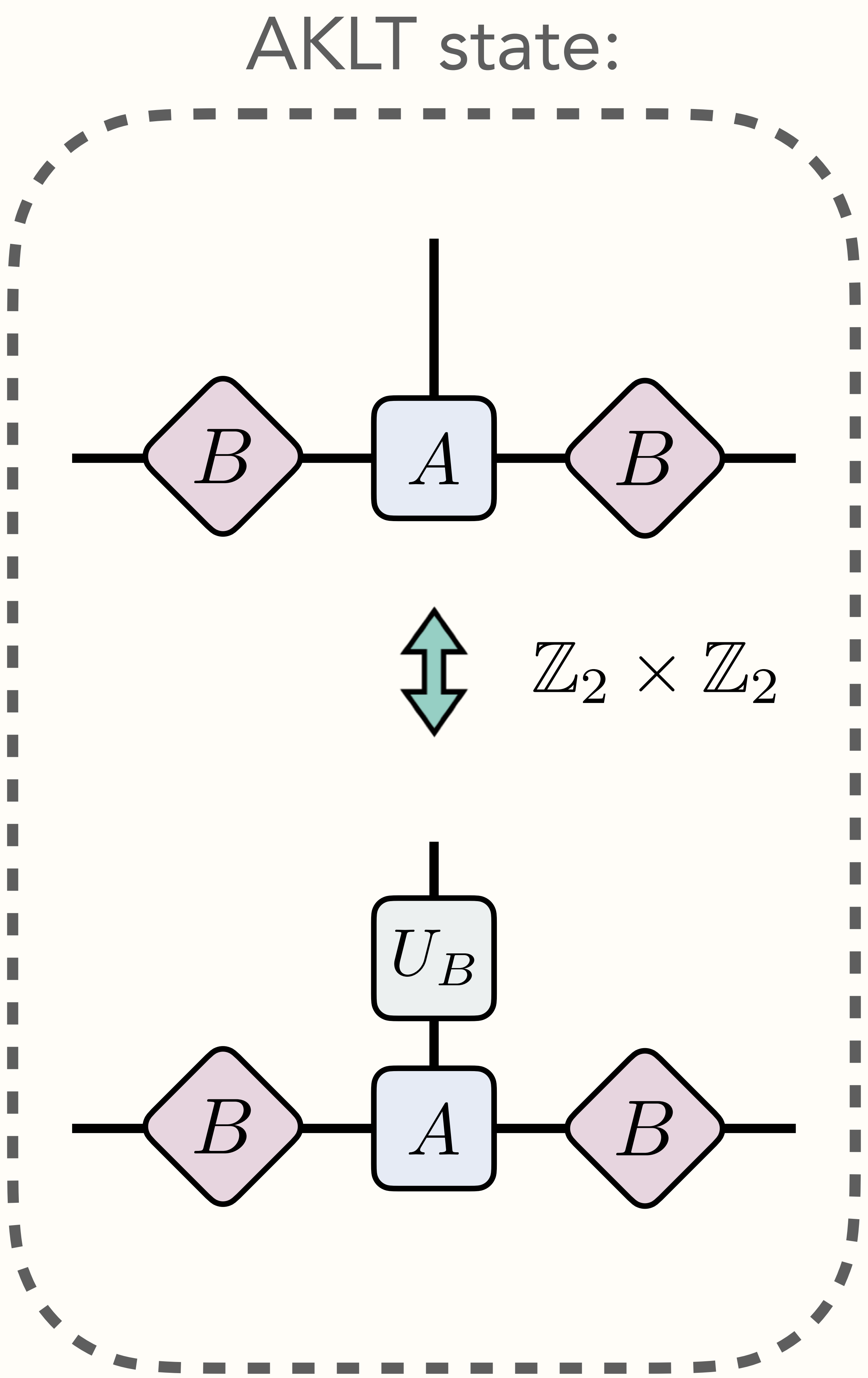

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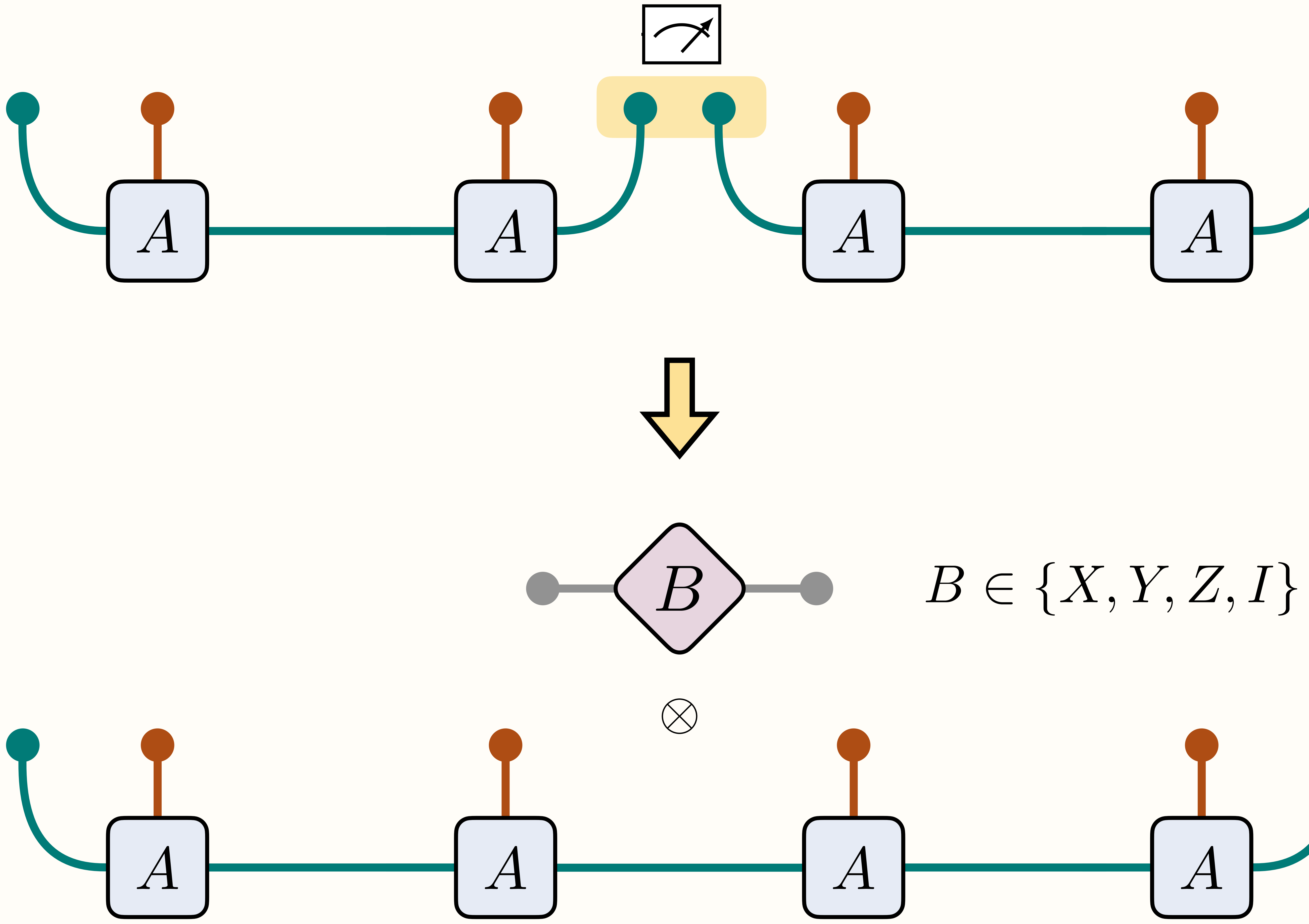
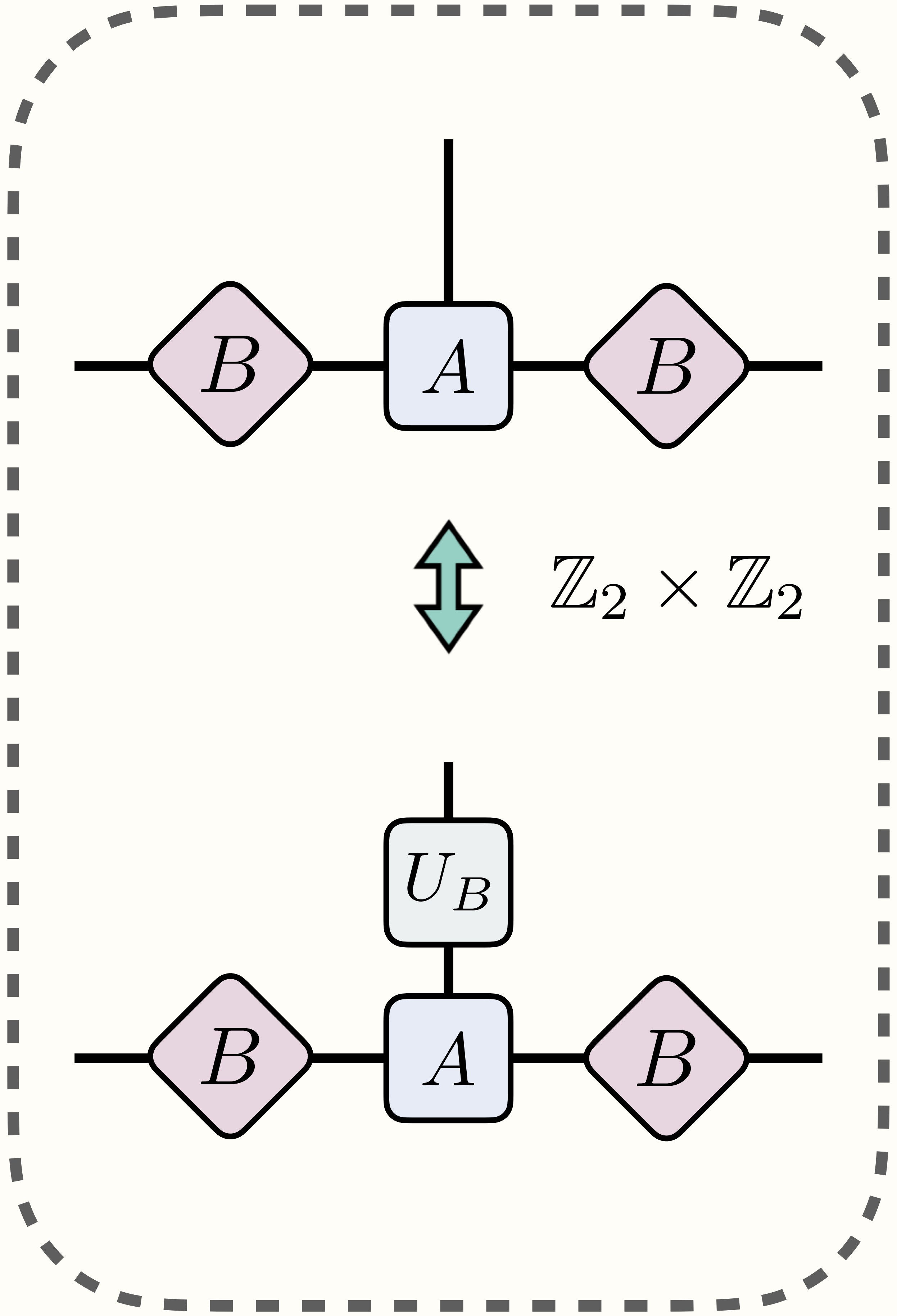
Ingredient #3: Defects can be corrected by leveraging symmetry

Measure in the Bell basis



AKLT state:



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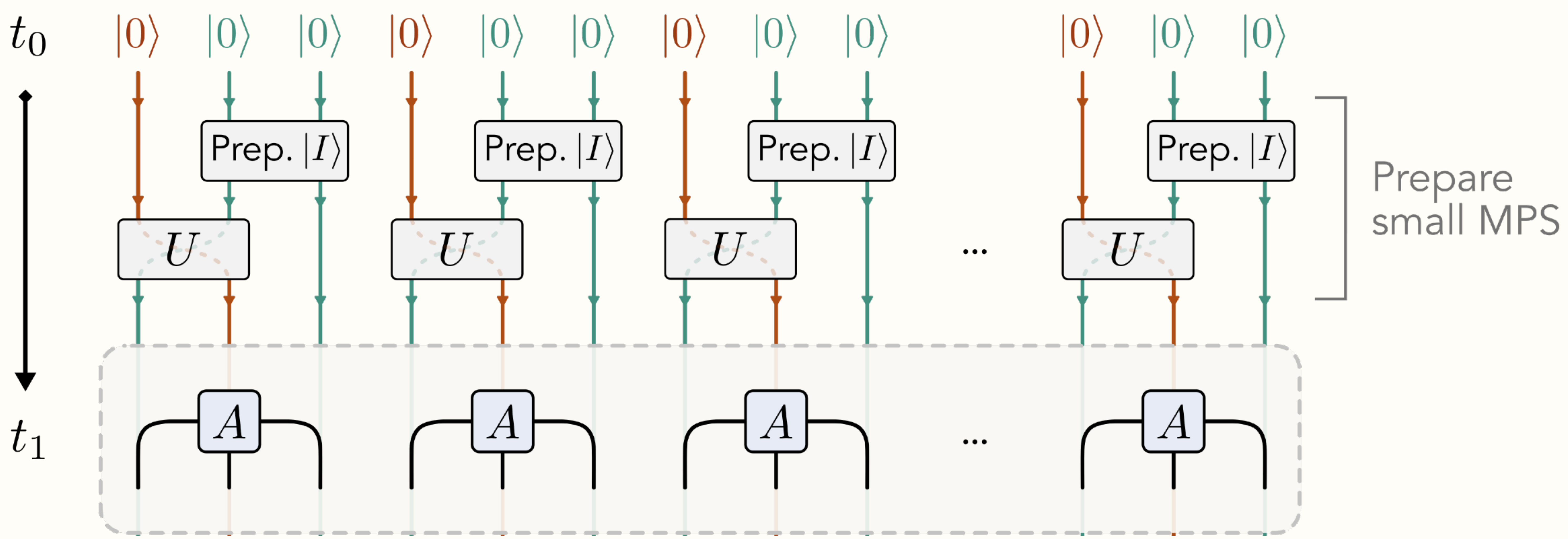

Putting it all together:

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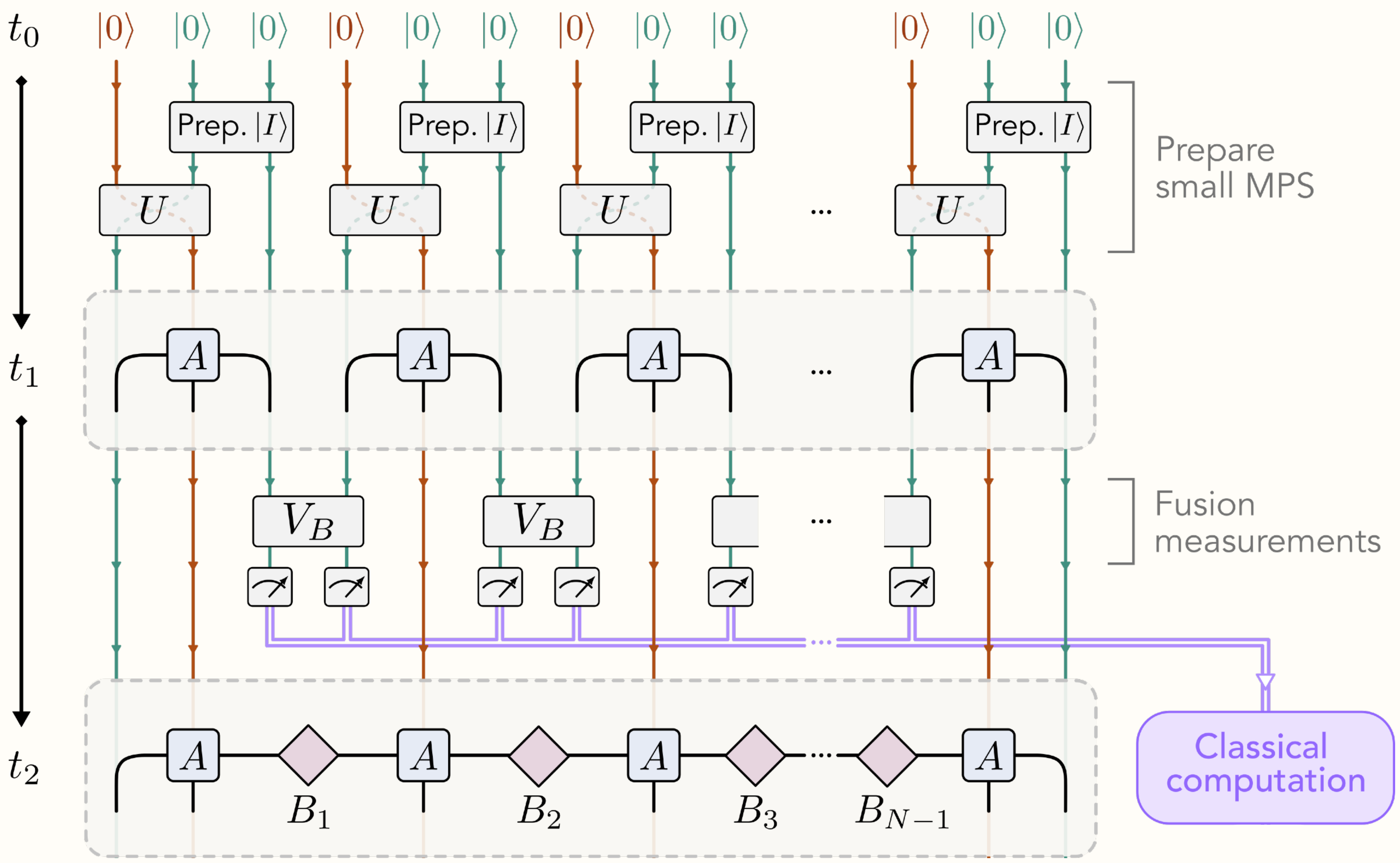

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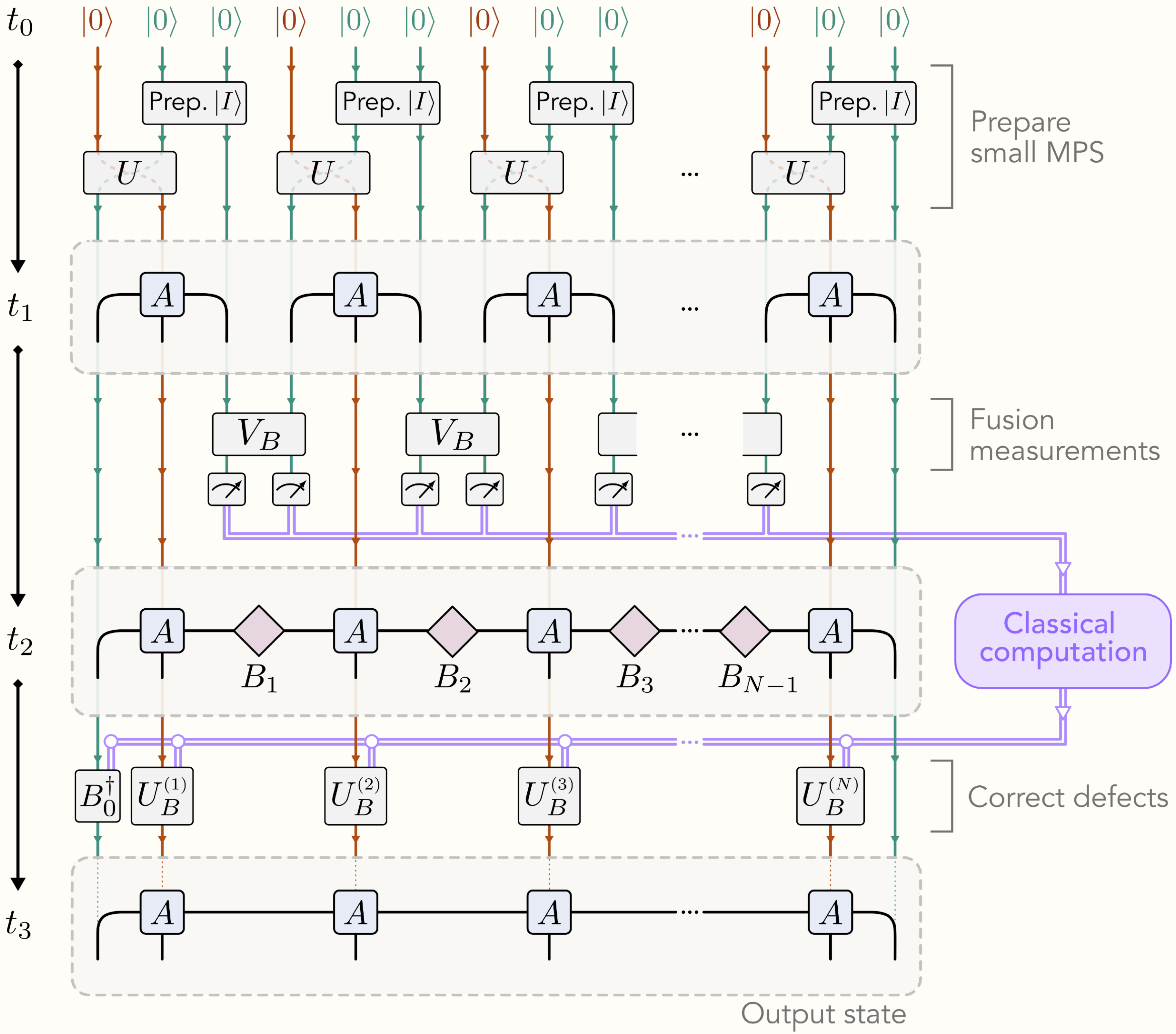
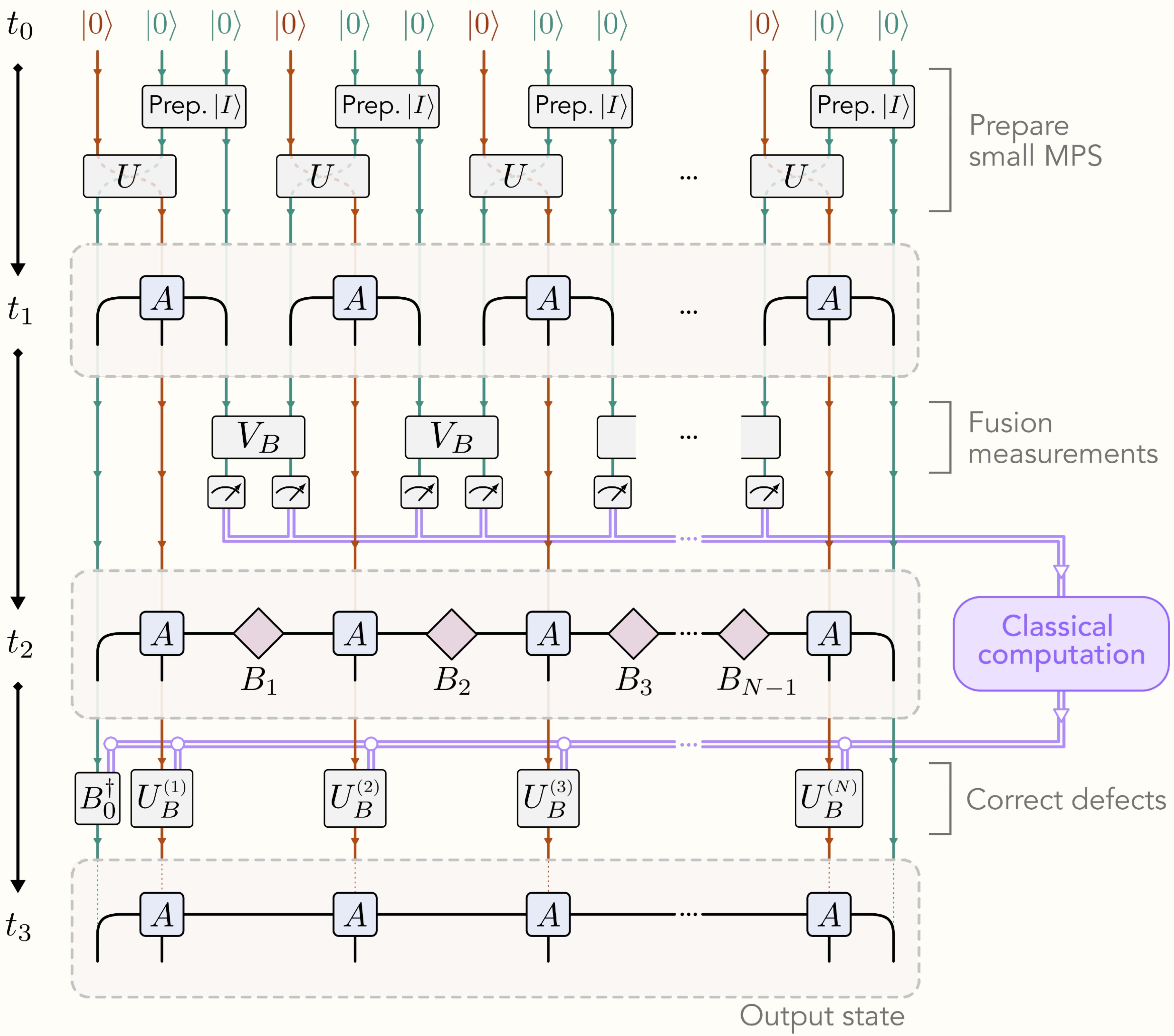


Putting it all together:

Deterministic + constant-depth

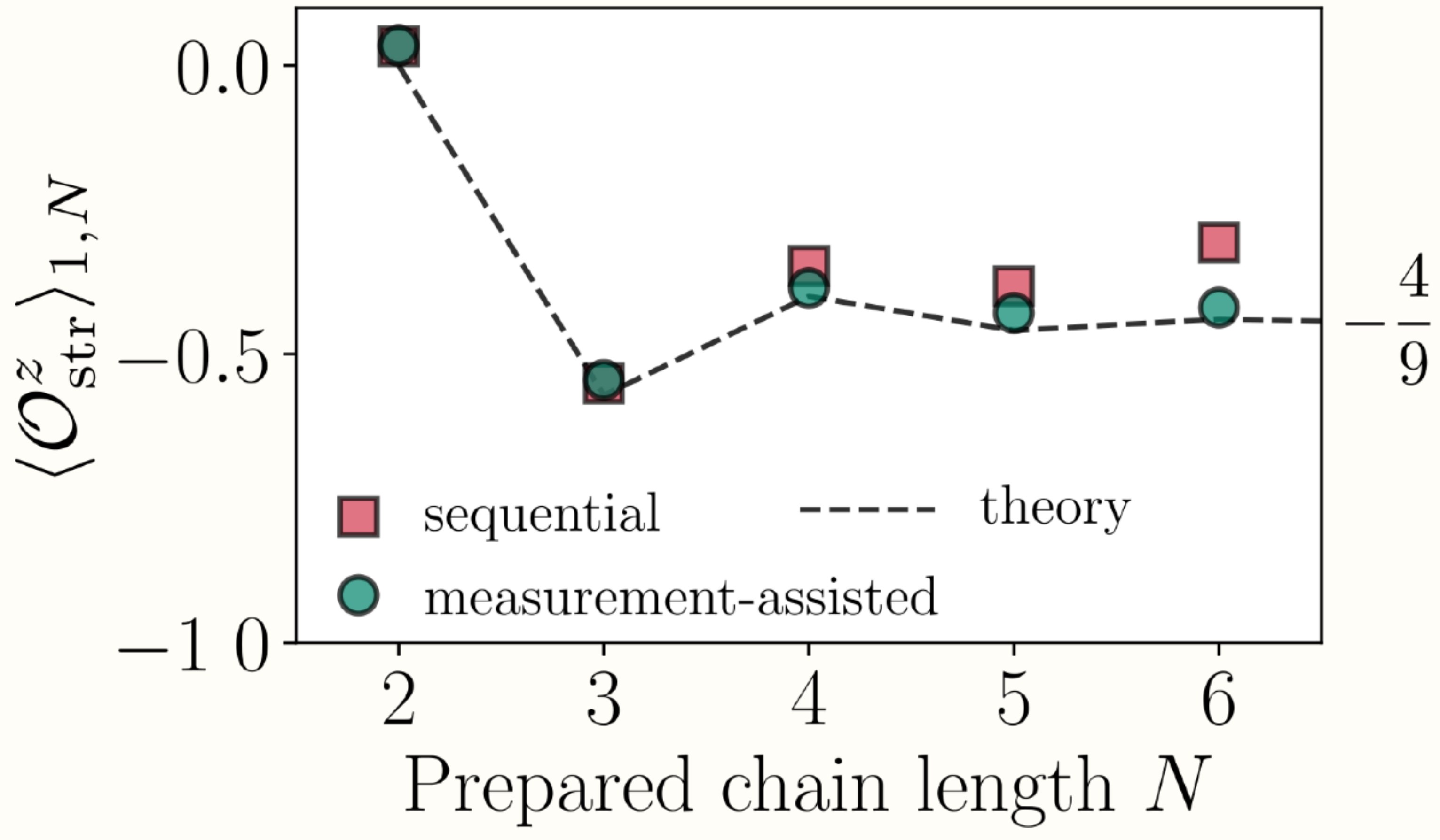
$T \sim O(1)$

$n_{\text{anc}} \sim O(N)$

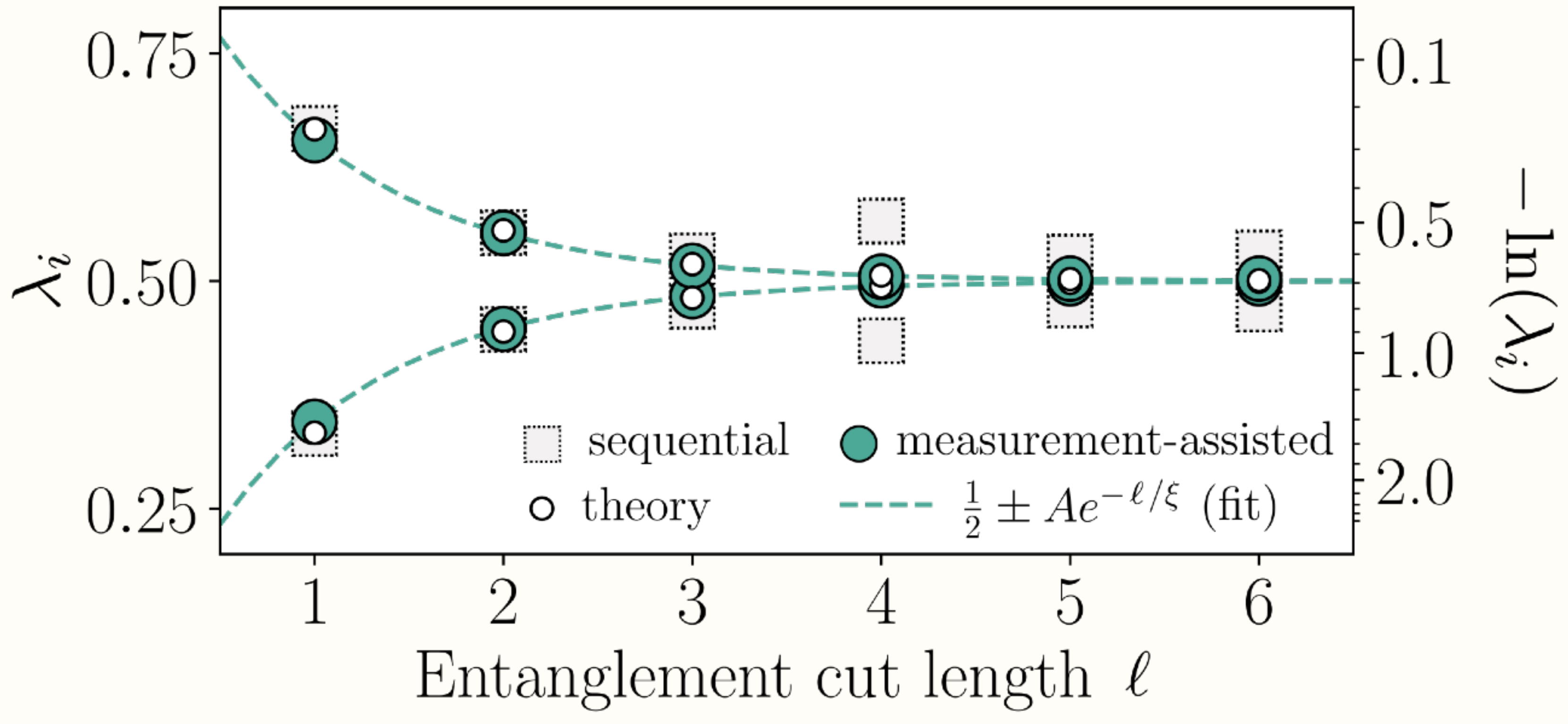


Preparation on an IBM Quantum processor

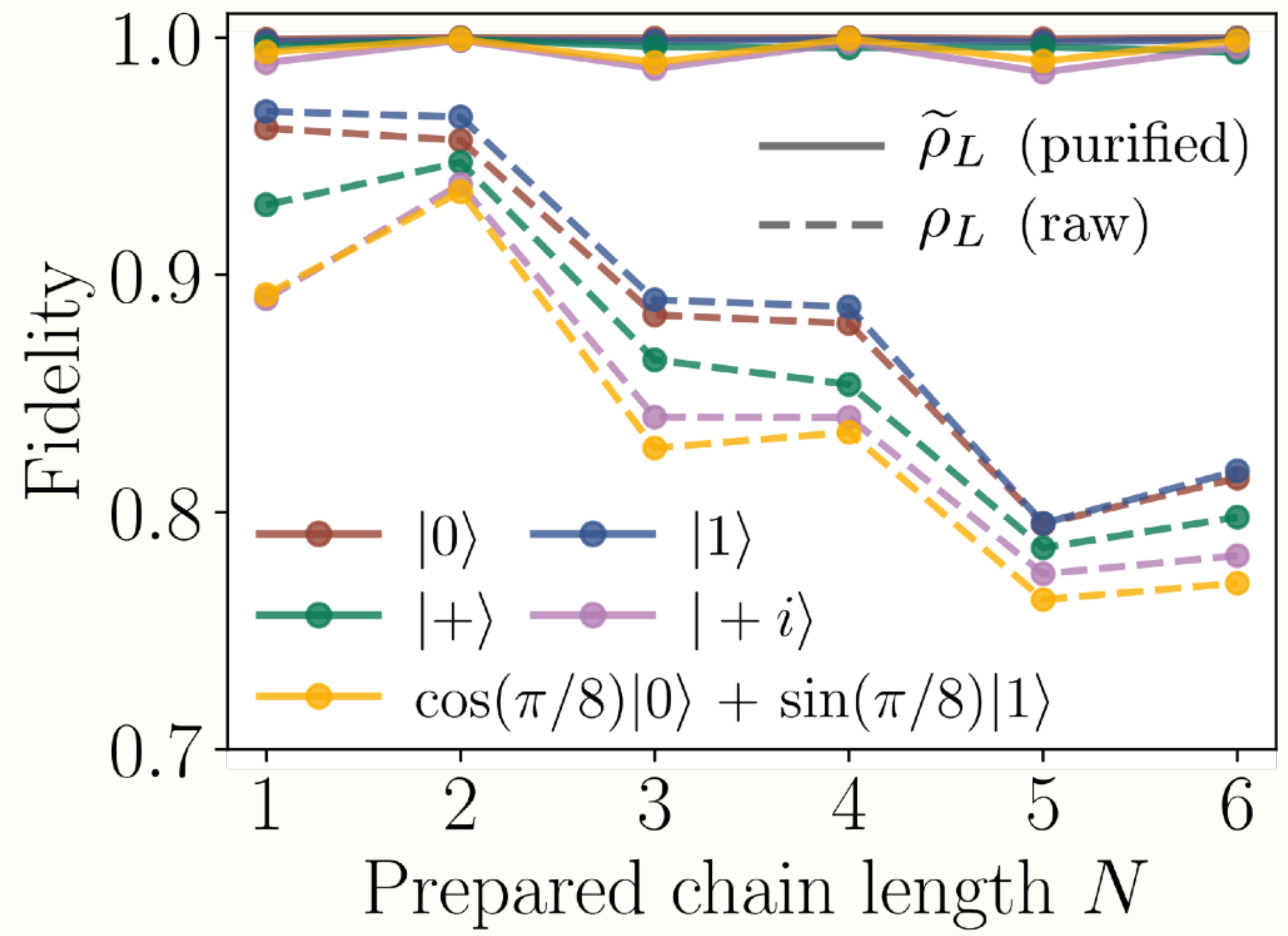
String order



Entanglement spectra



Quantum teleportation



and more…

See Smith et al, PRX Quantum 4, 020315 (2023) for details

What other matrix product states are preparable with this algorithm?

Our work:

Smith, Khan, Clark, Wei, and Girvin, PRX Quantum 5, 030344 (2024)

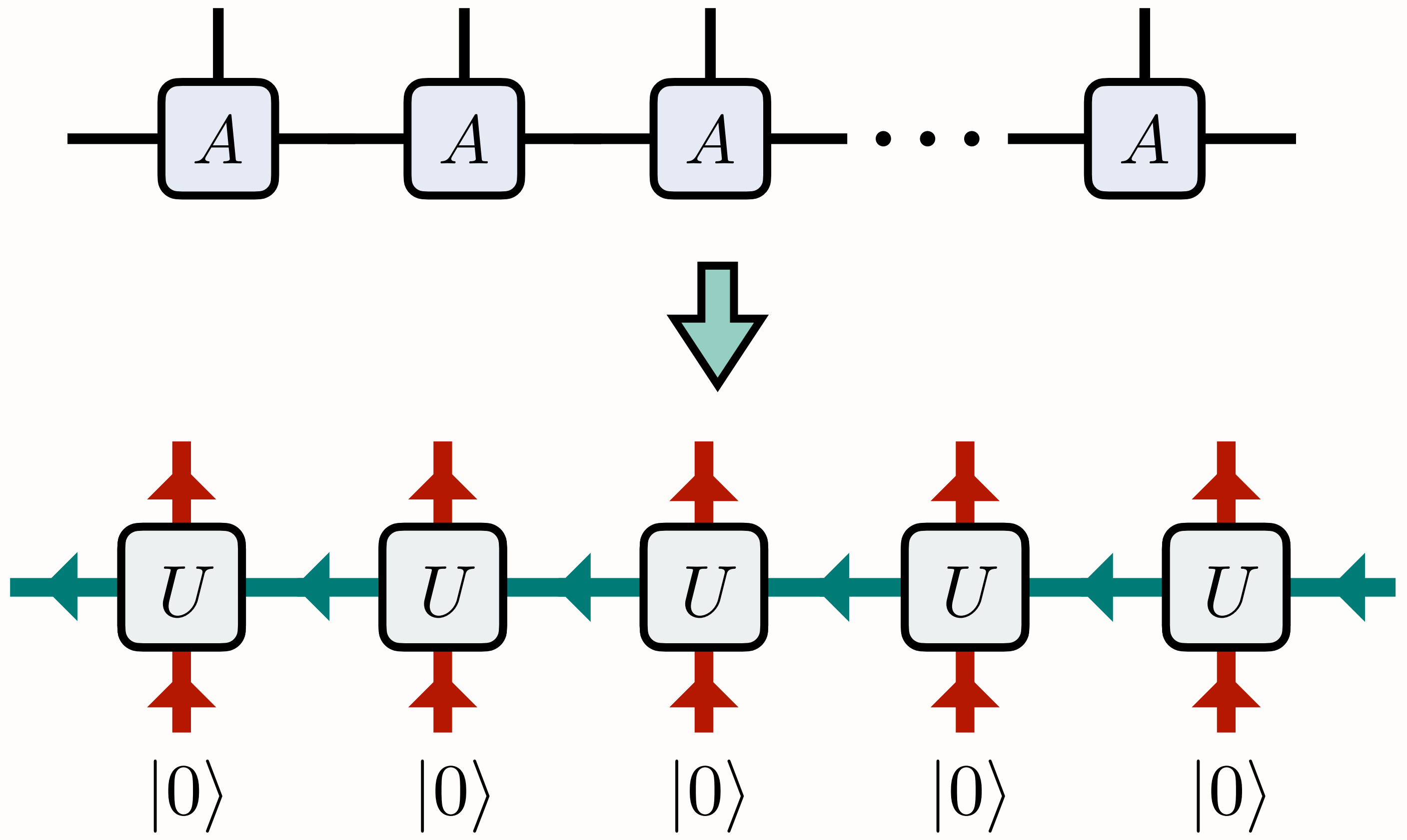
Related work:

Stephen and Hart, arXiv: 2404.16360 (2024)

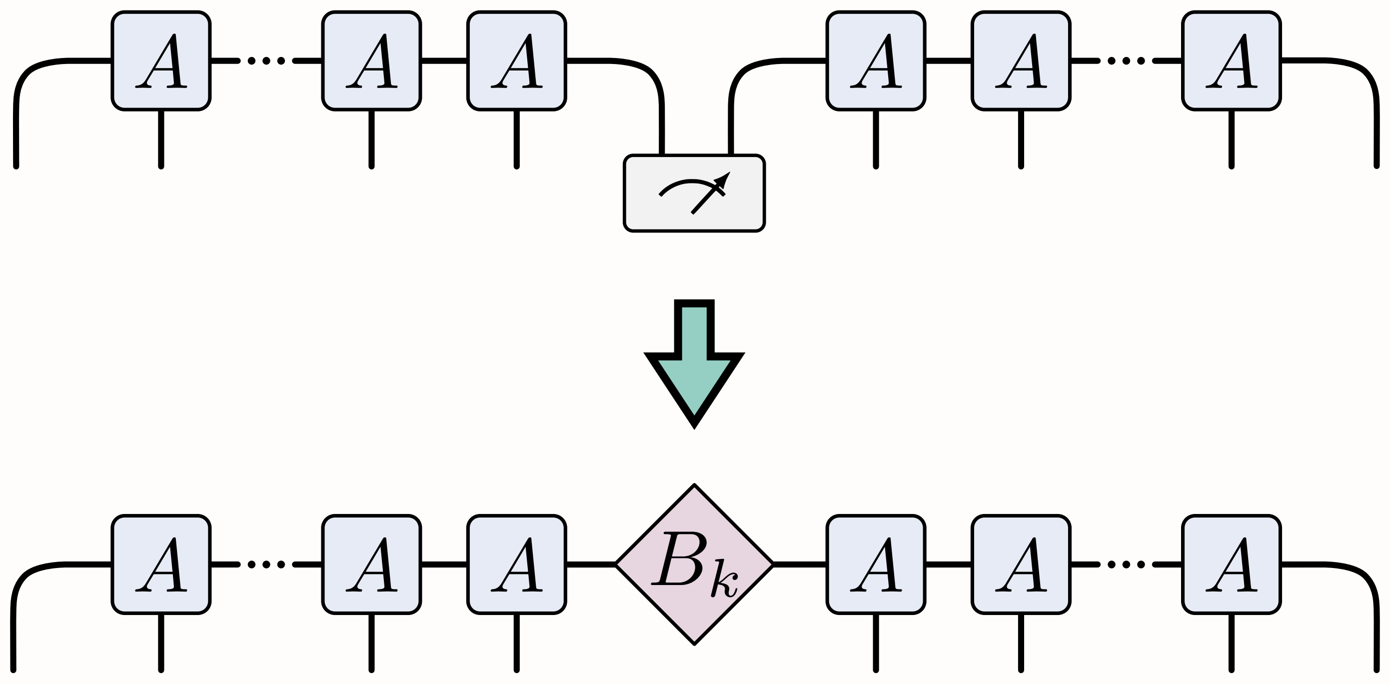
Sahay and Verresen, arXiv: 2404.16753 (2024)

Zhang, Gopalakrishnan, Styliaris, arXiv: 2405.09615 (2024)

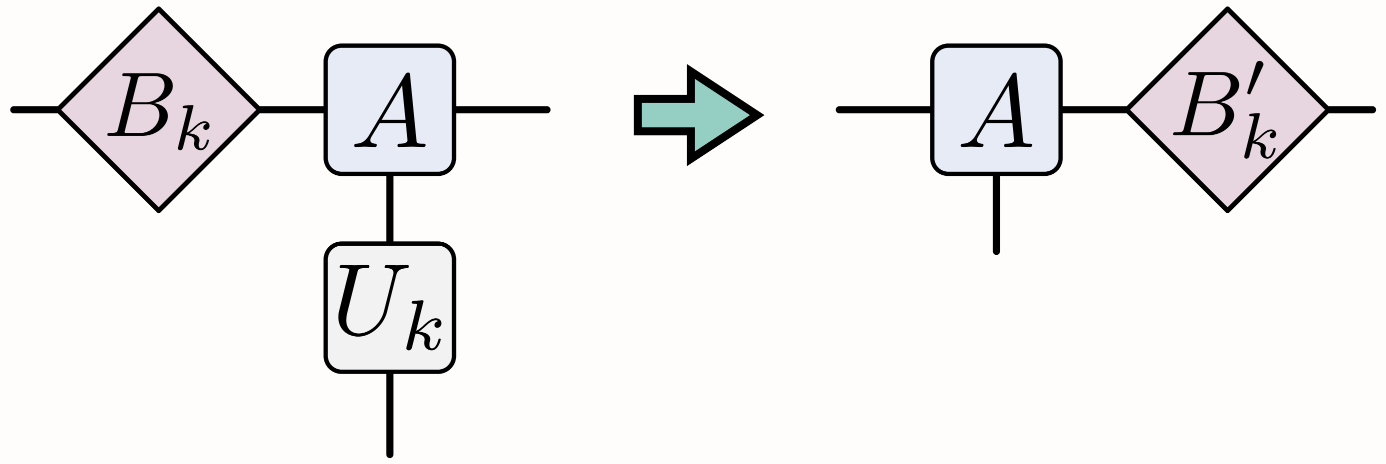
① Sequential preparation



② Fusion measurements



③ Operator pushing



Smith, Khan, Clark, Wei, and Girvin, PRX Quantum 5, 030344 (2024)

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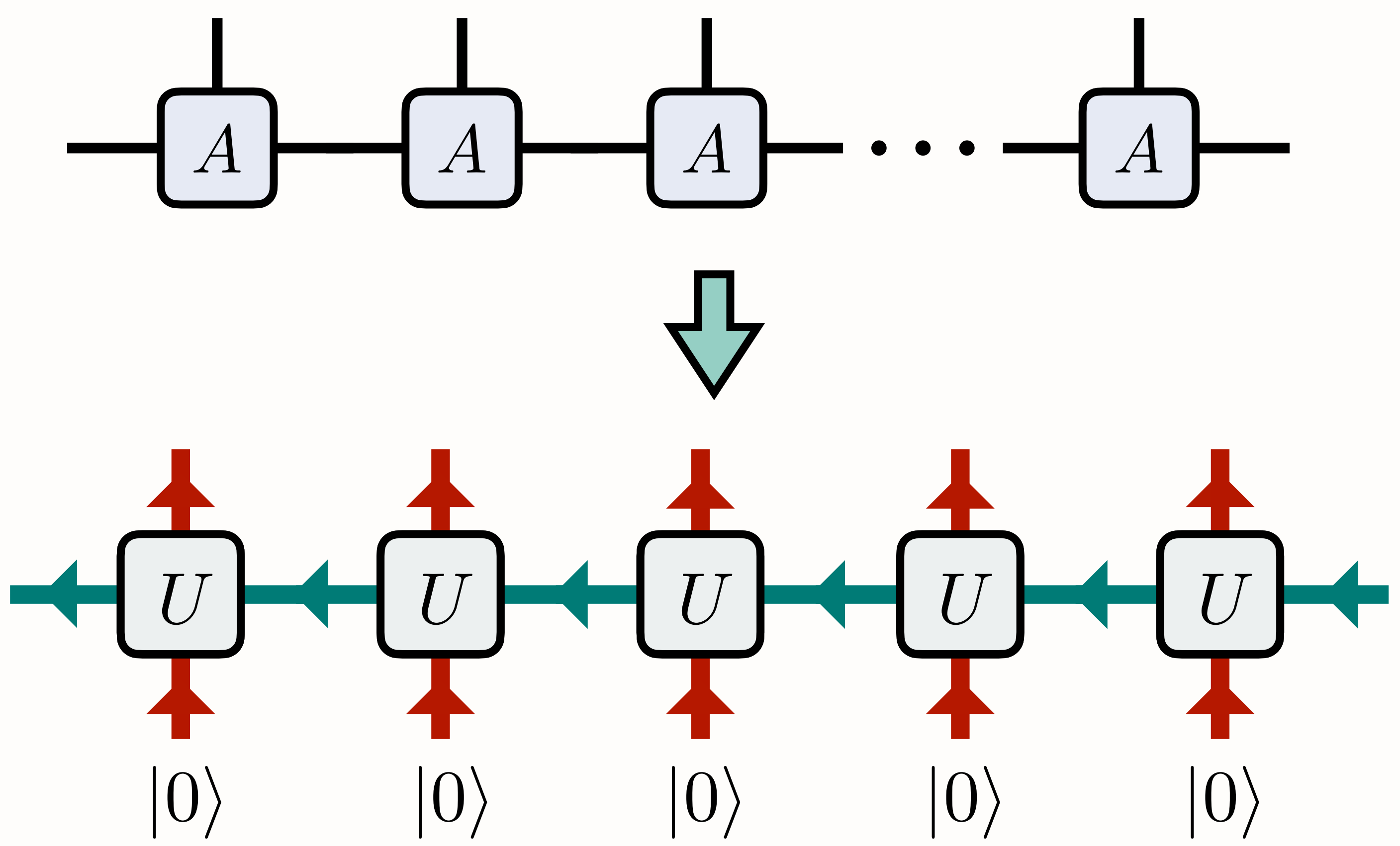

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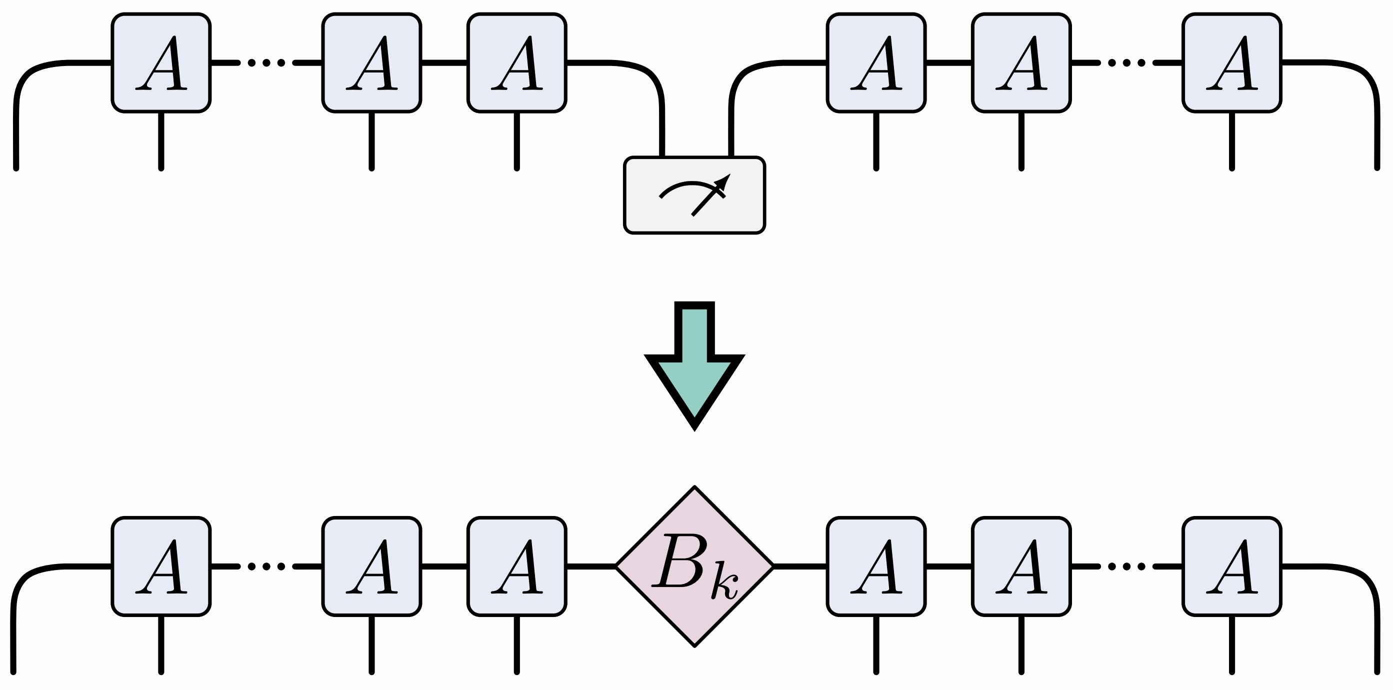

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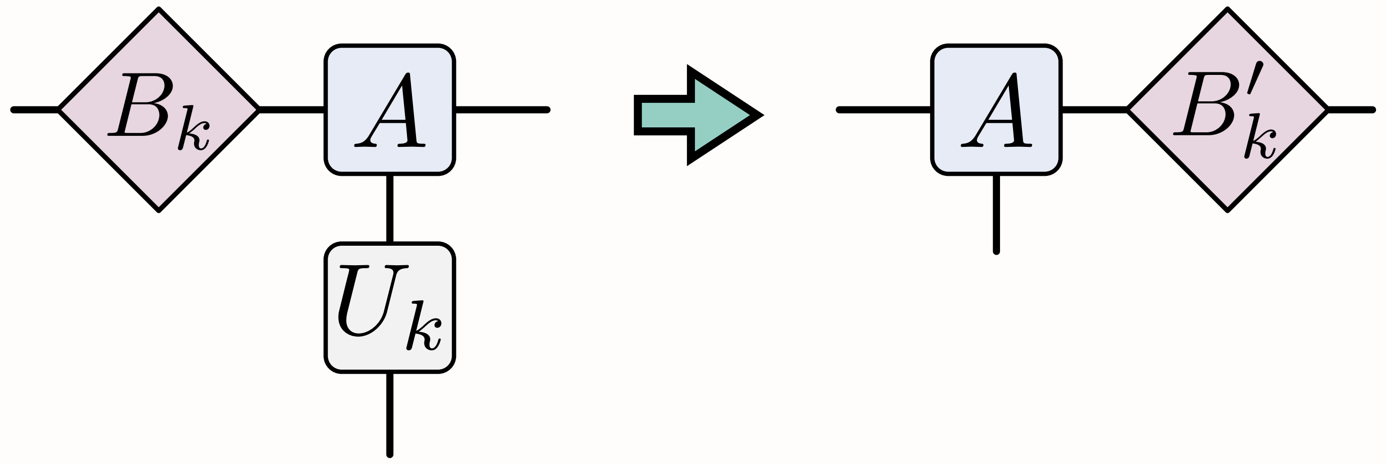

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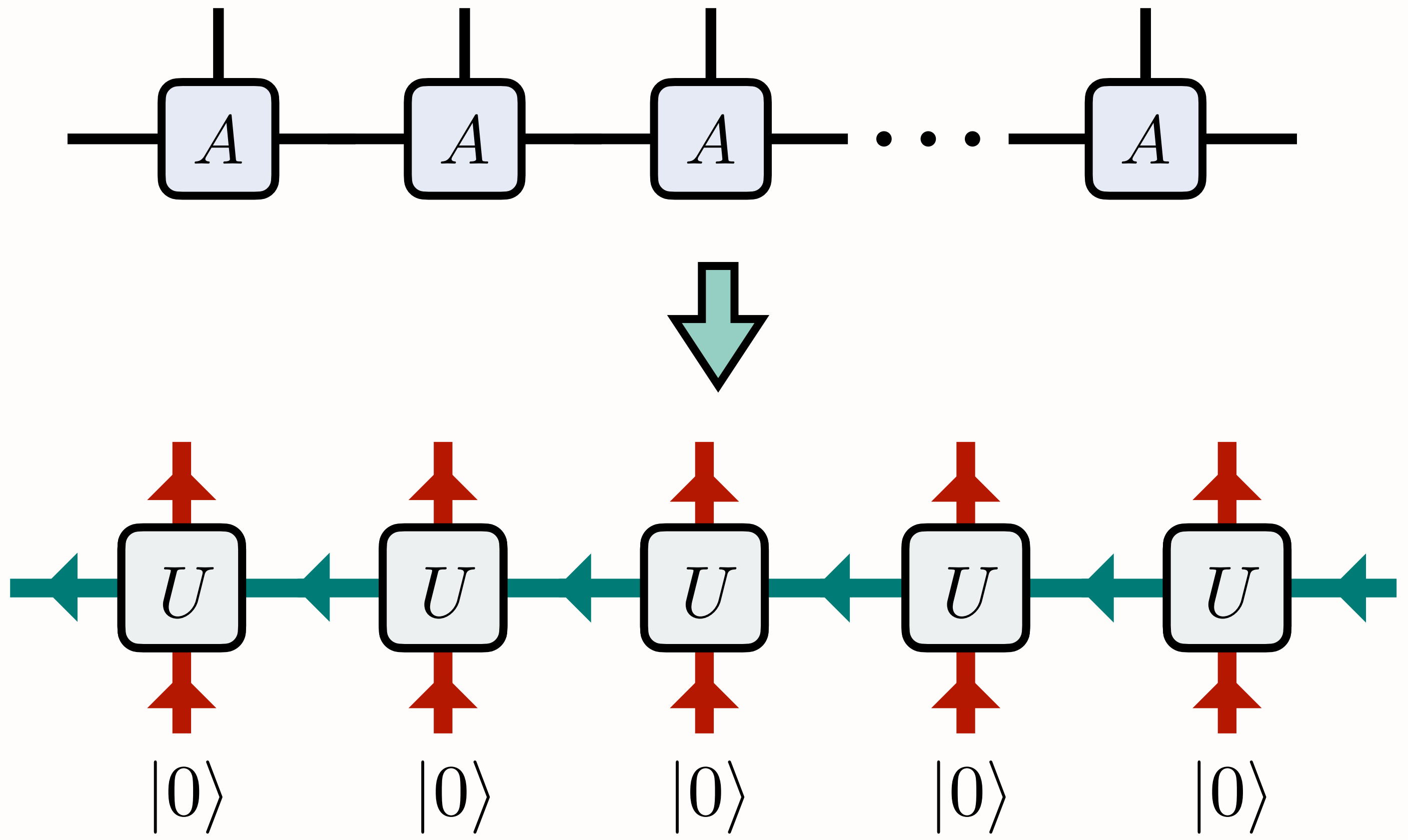

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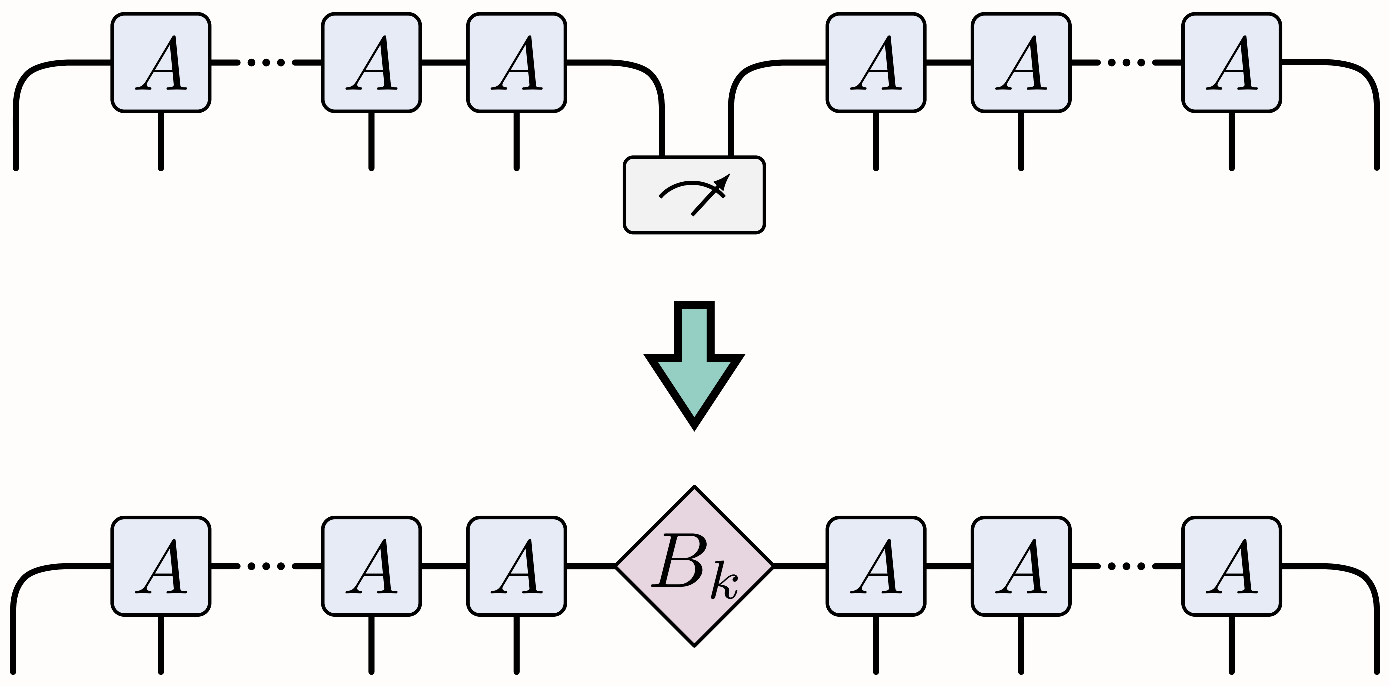

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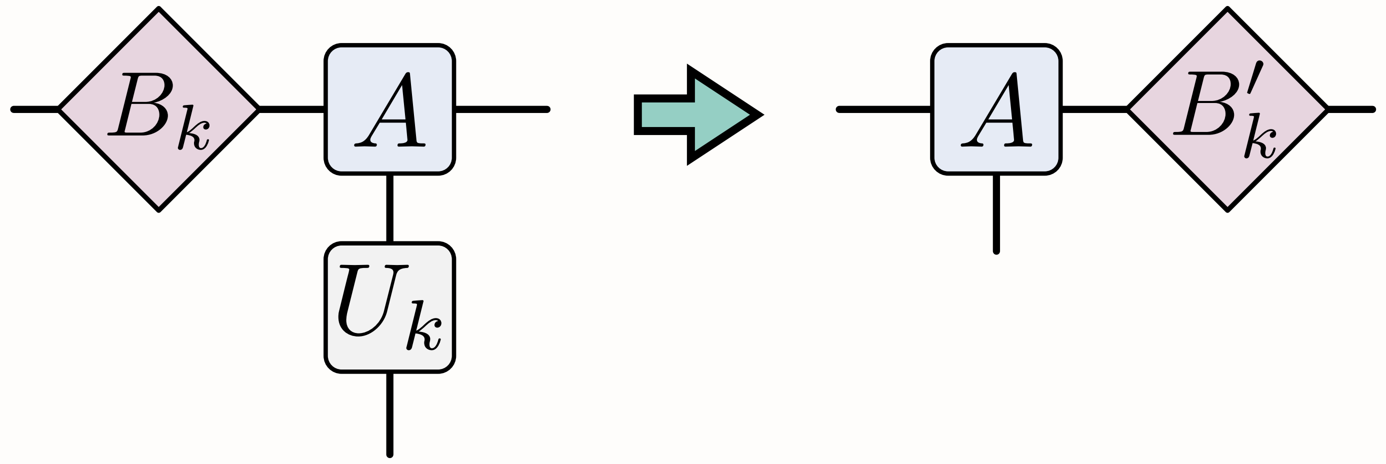

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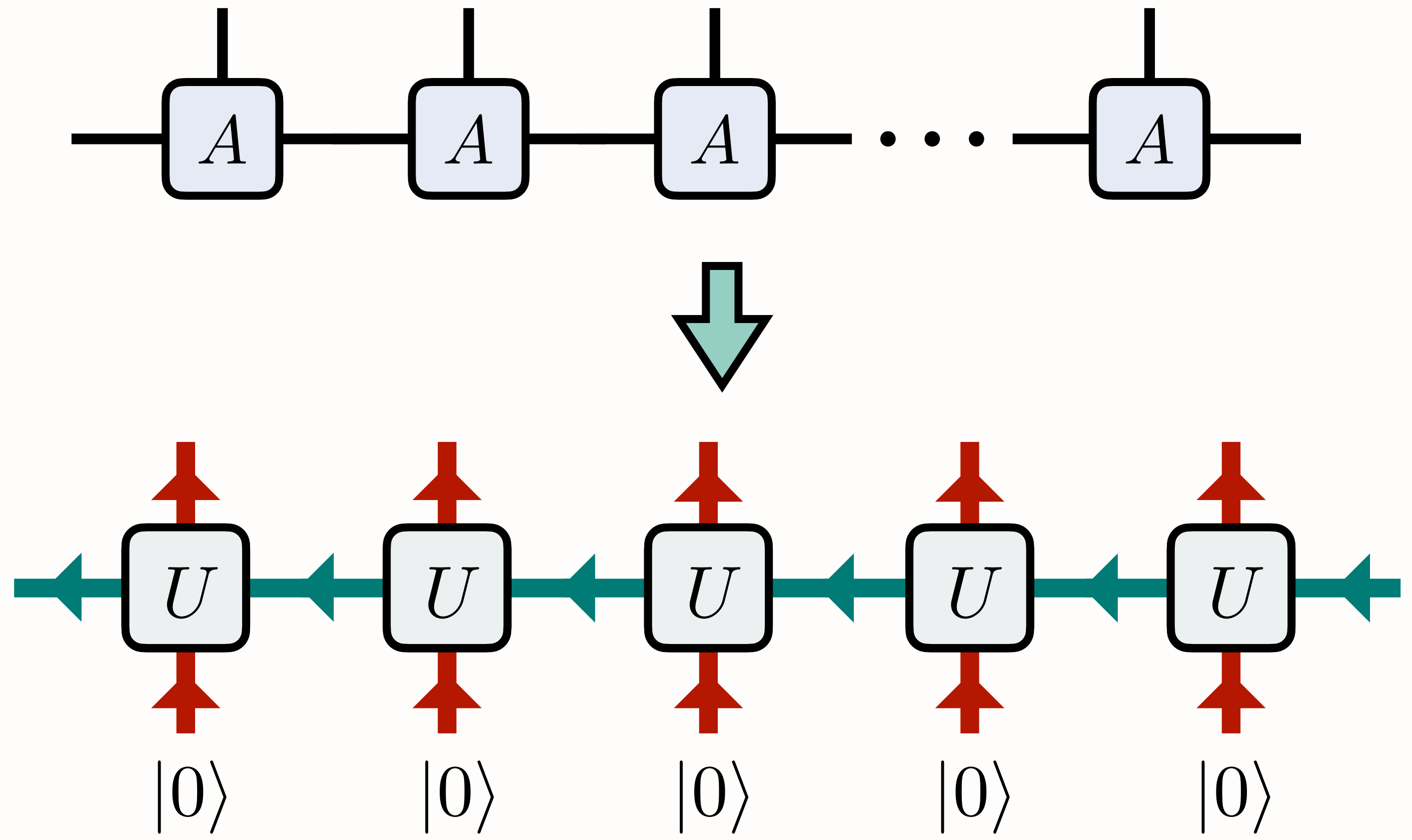

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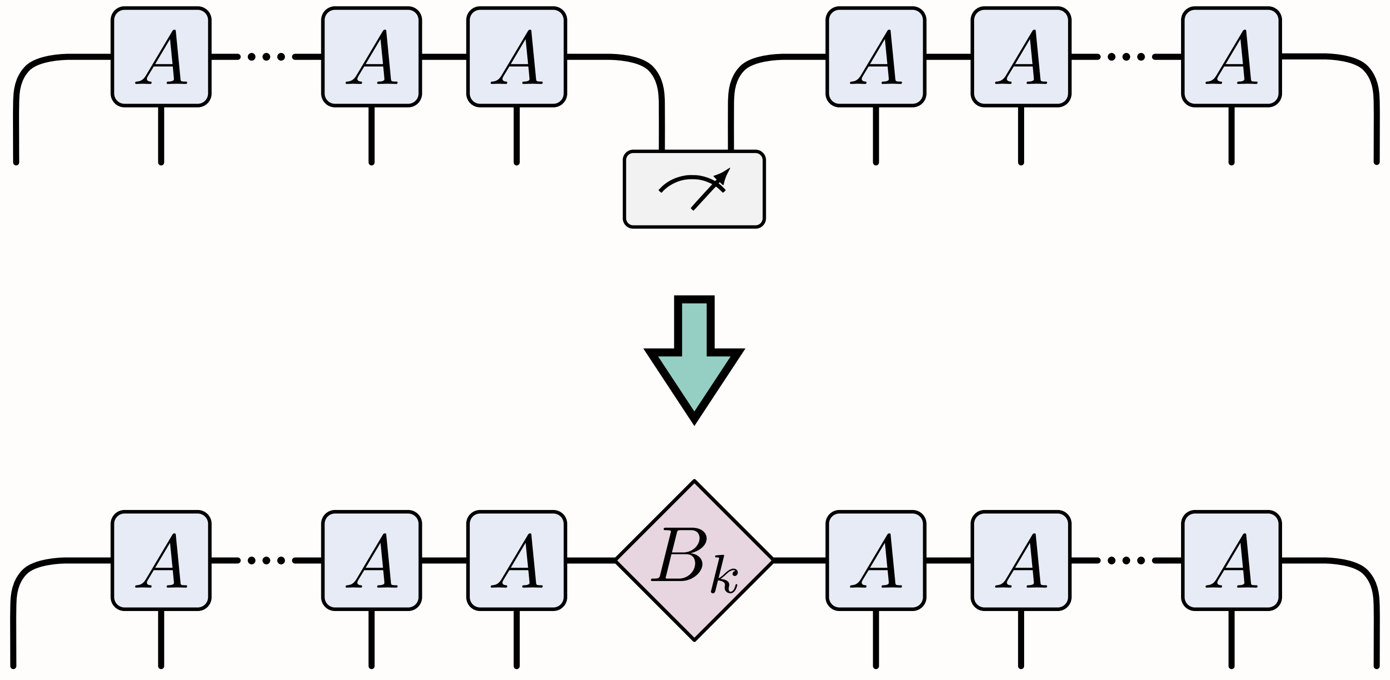

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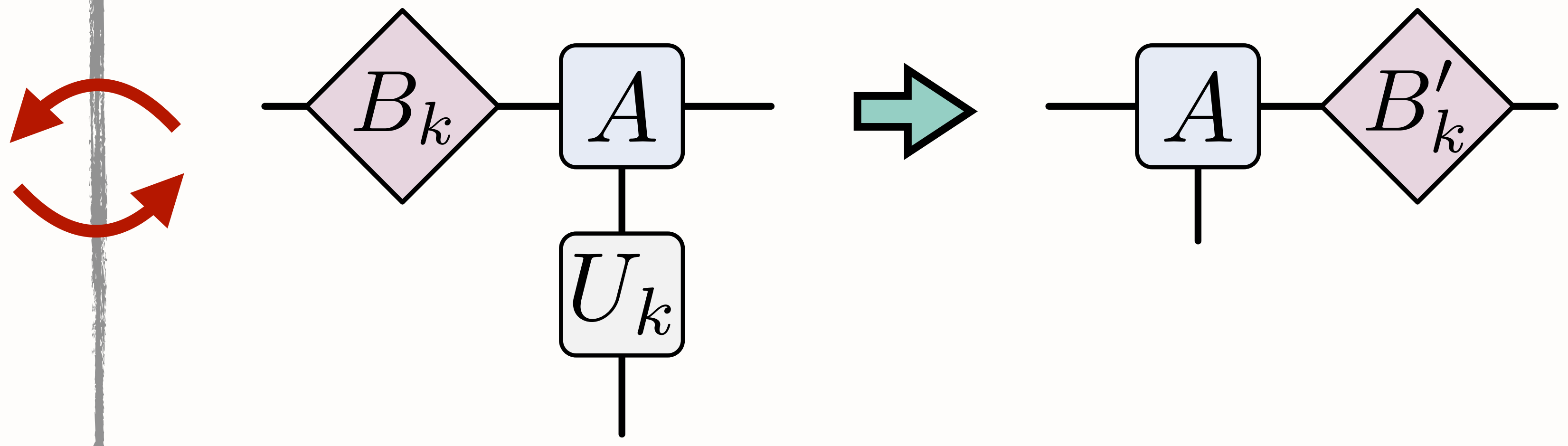

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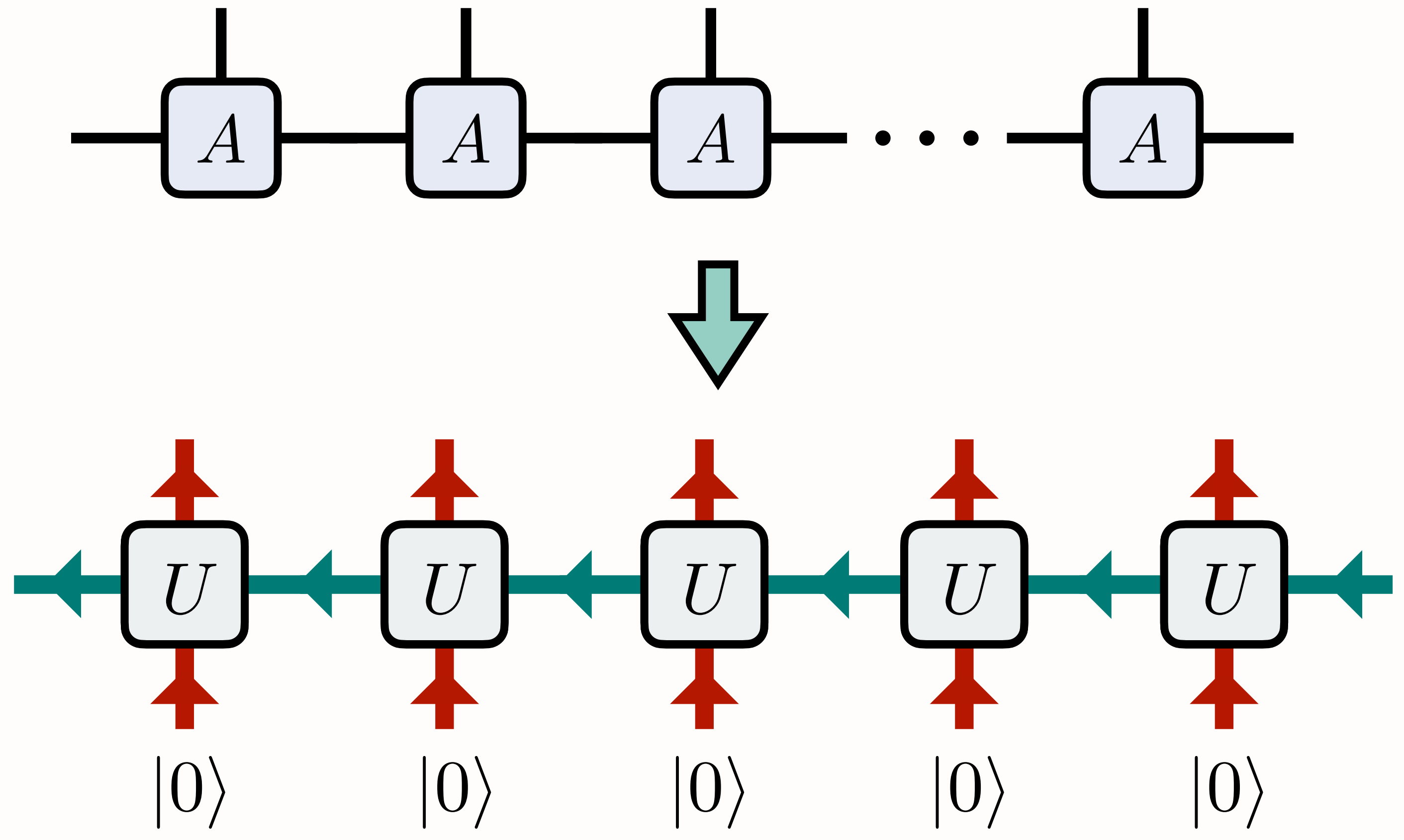

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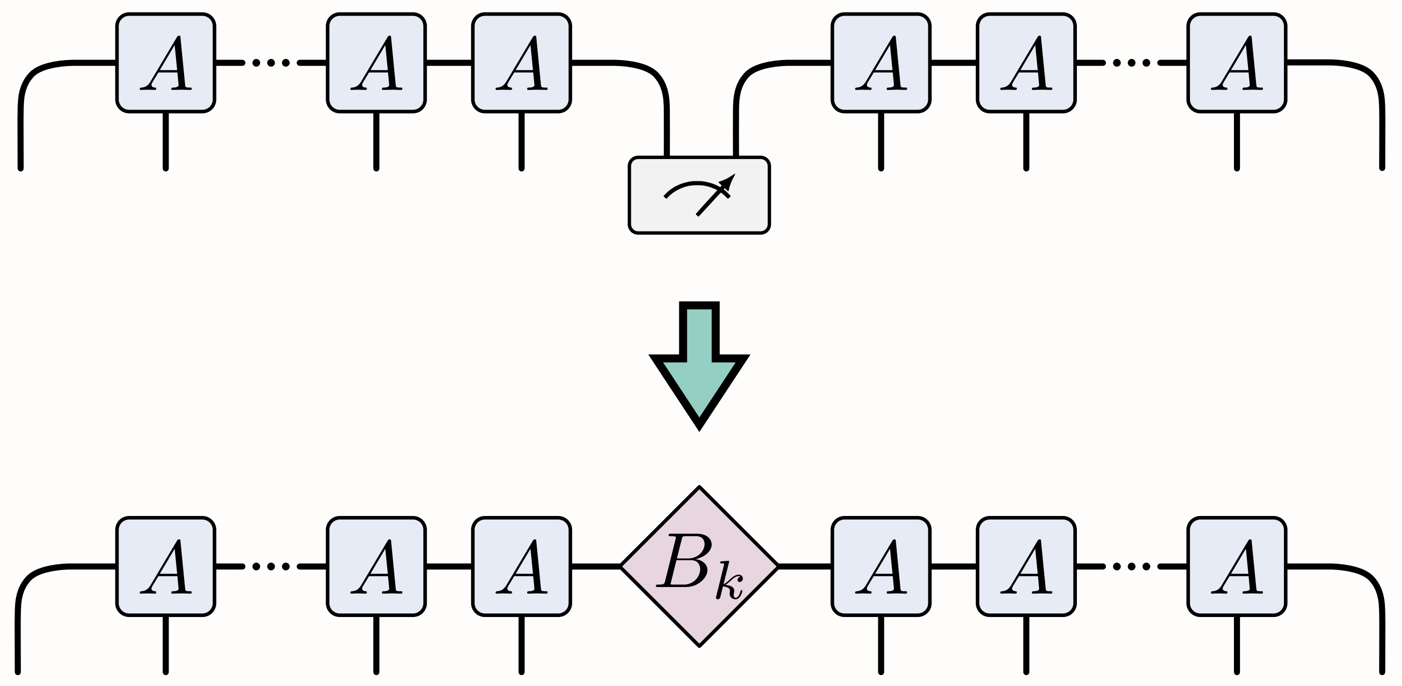
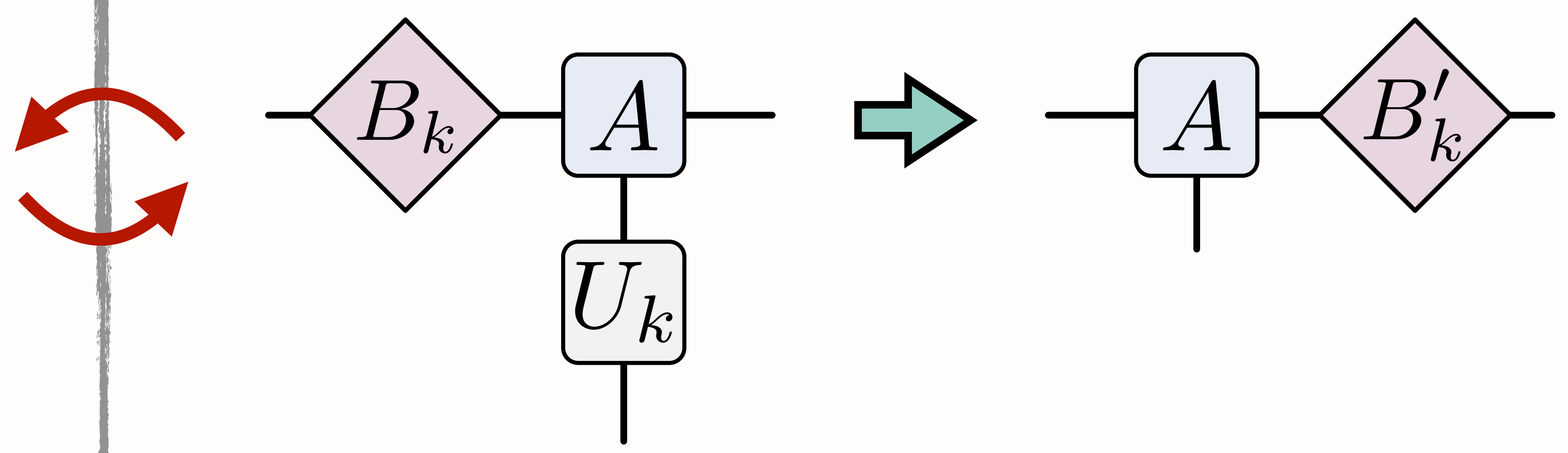


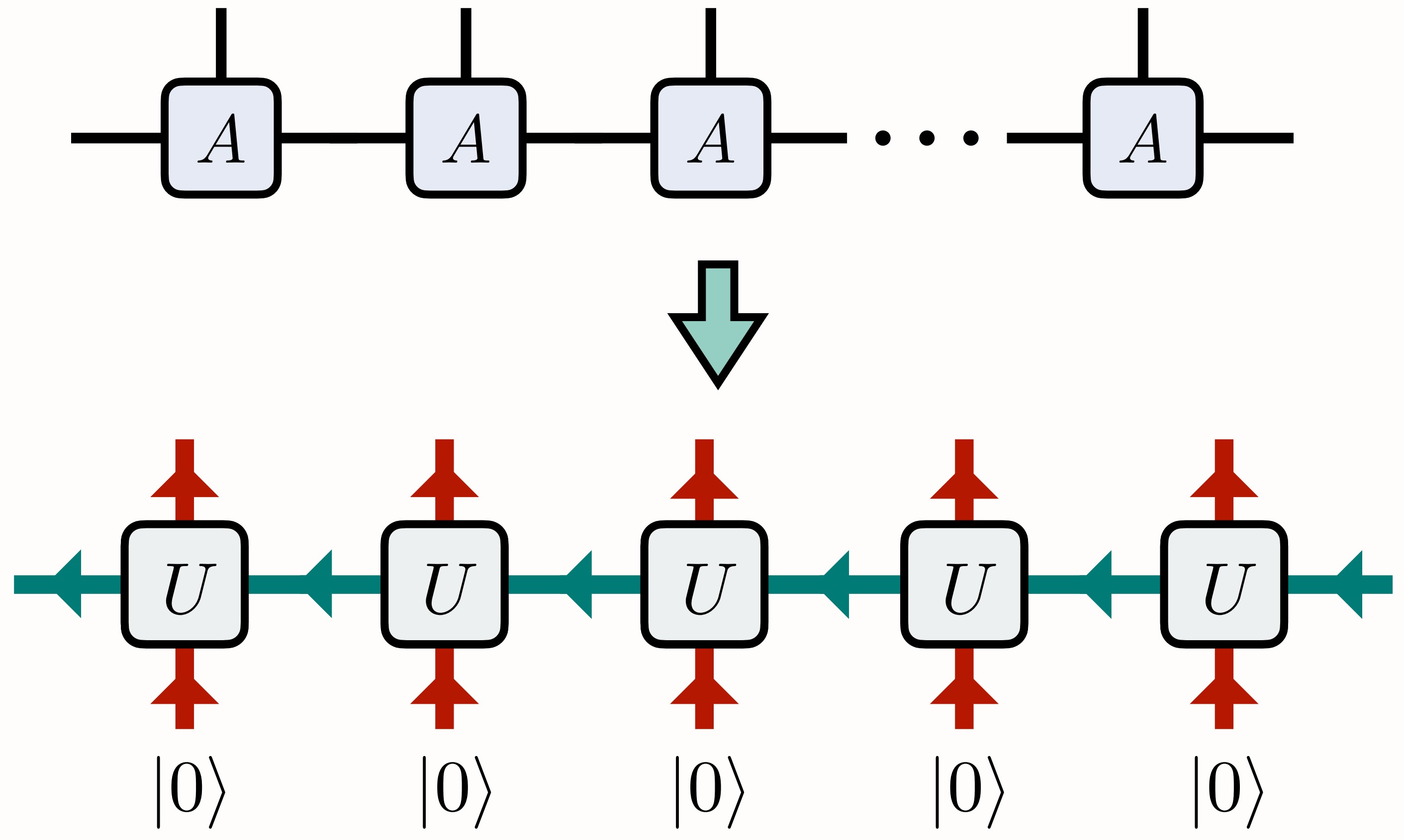
③ Operator pushing ?



- Challenge: need a complete measurement basis (dim D^2) that is "pushable."

Smith, Khan, Clark, Wei, and Girvin, PRX Quantum 5, 030344 (2024)

① Sequential preparation



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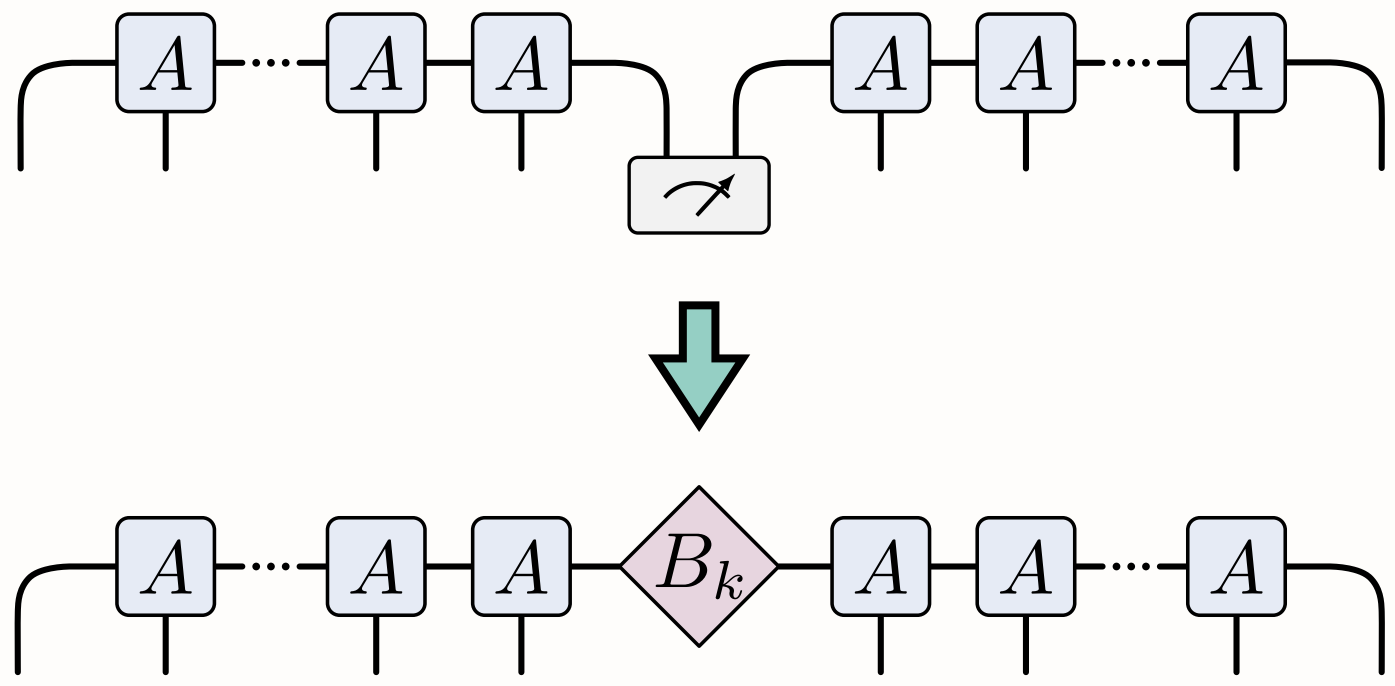

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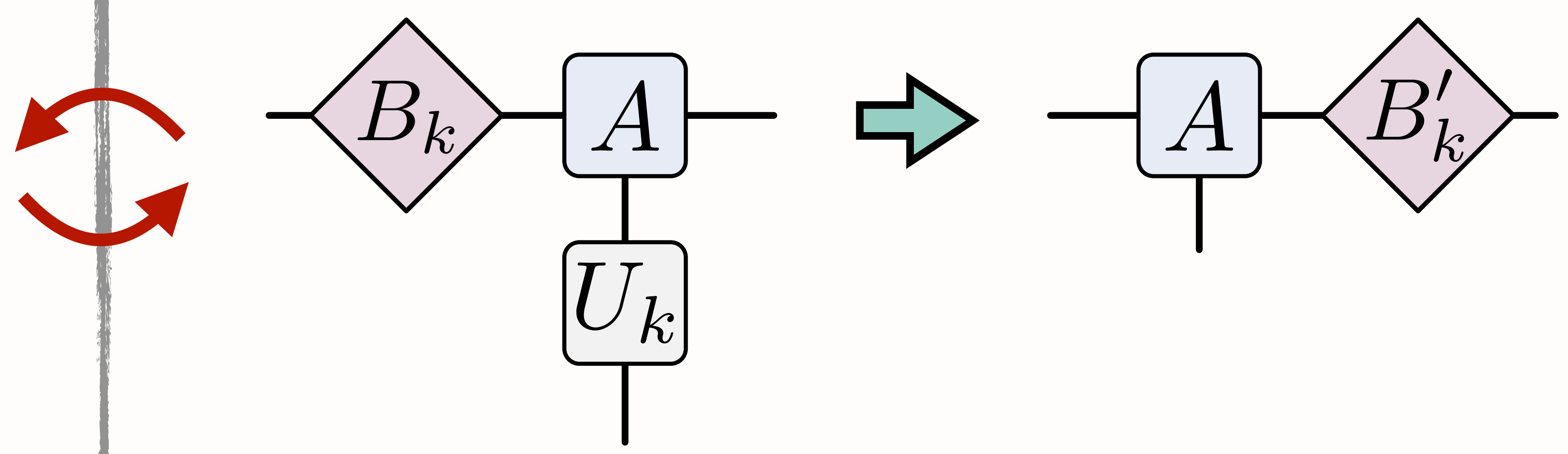
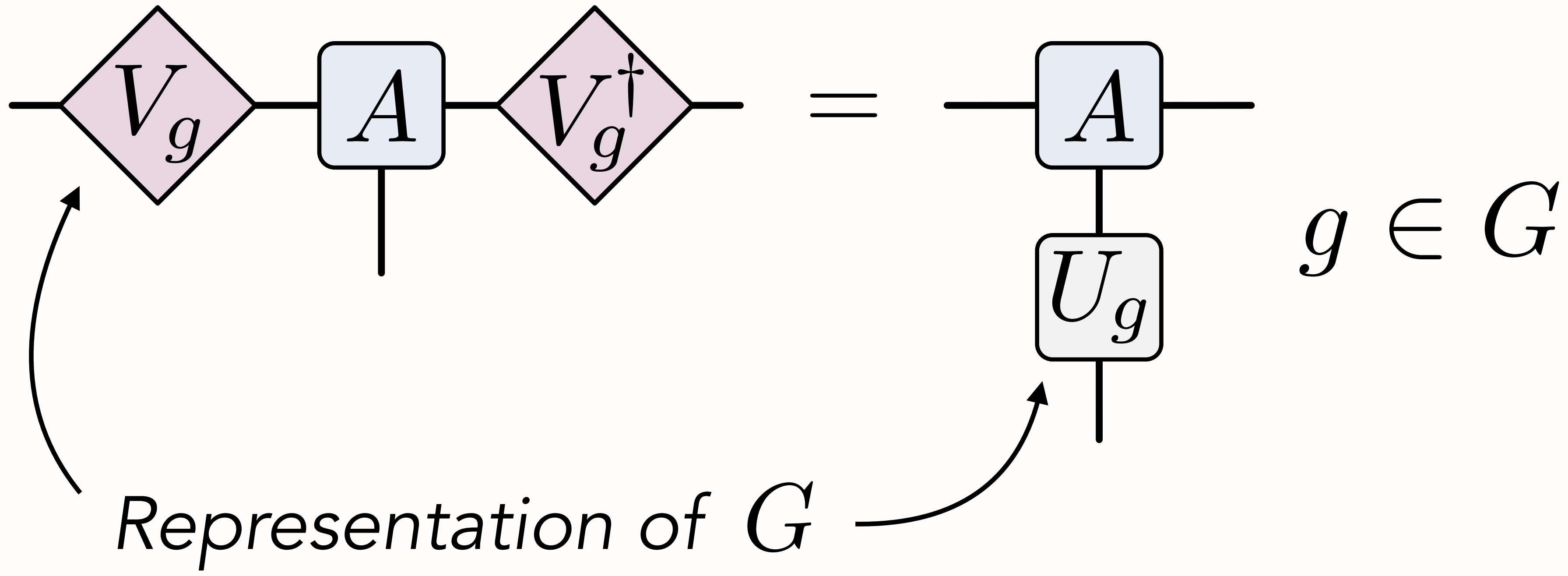


- Challenge: need a complete measurement basis (dim D^2) that is "pushable."

- One special scenario: on-site symmetry guarantees pushing relations



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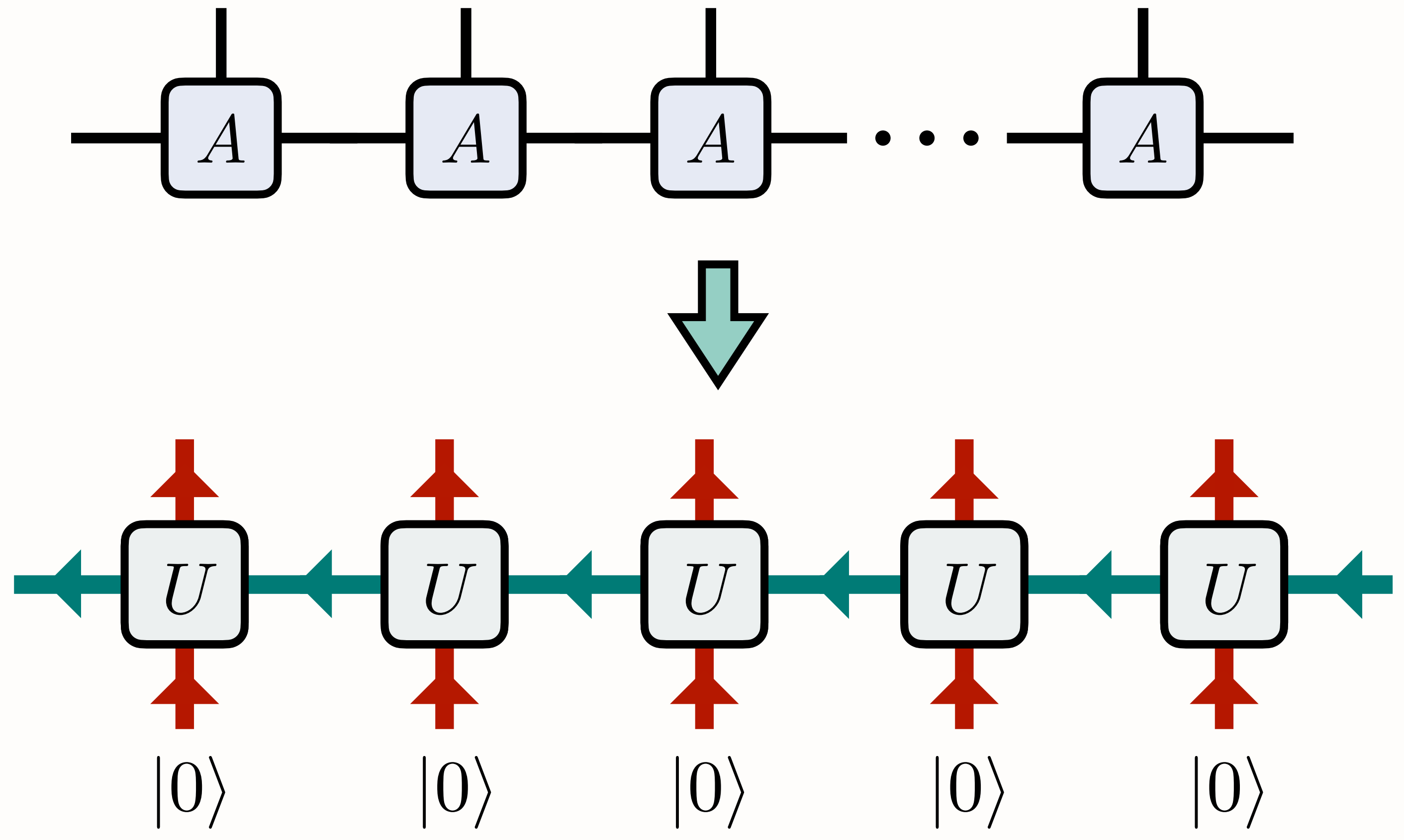

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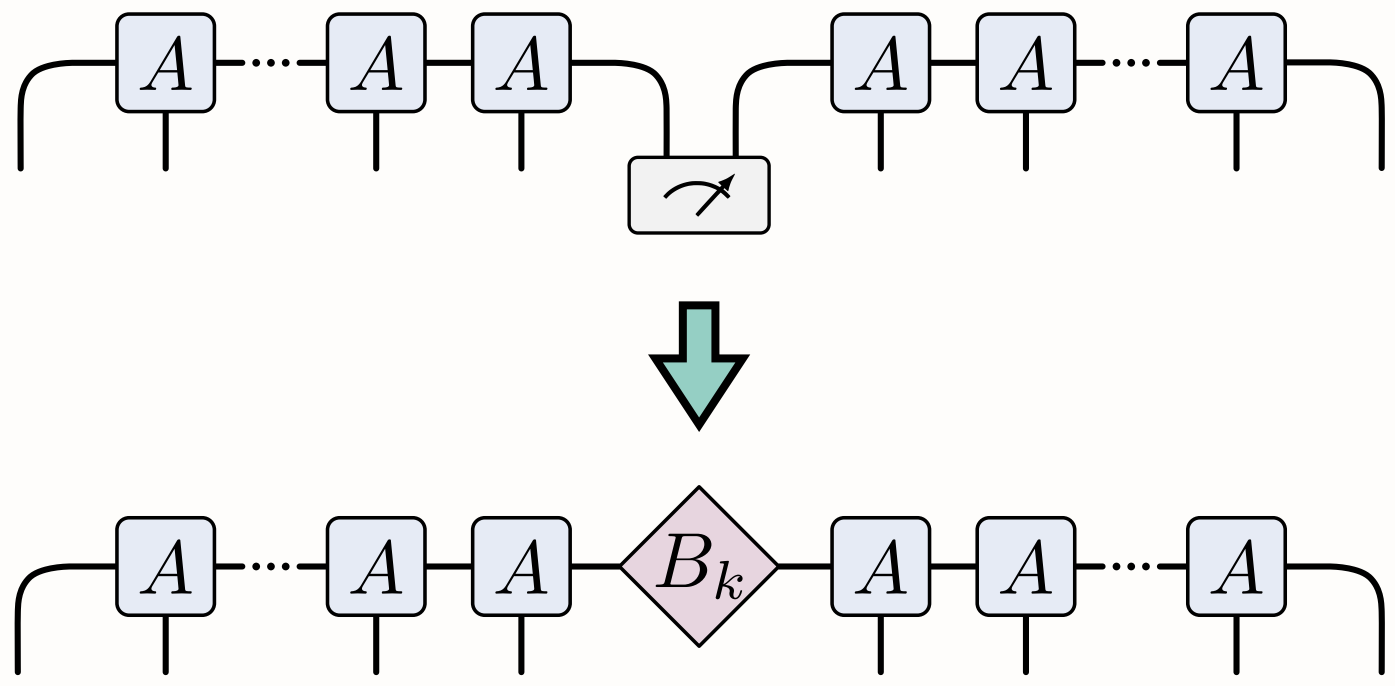

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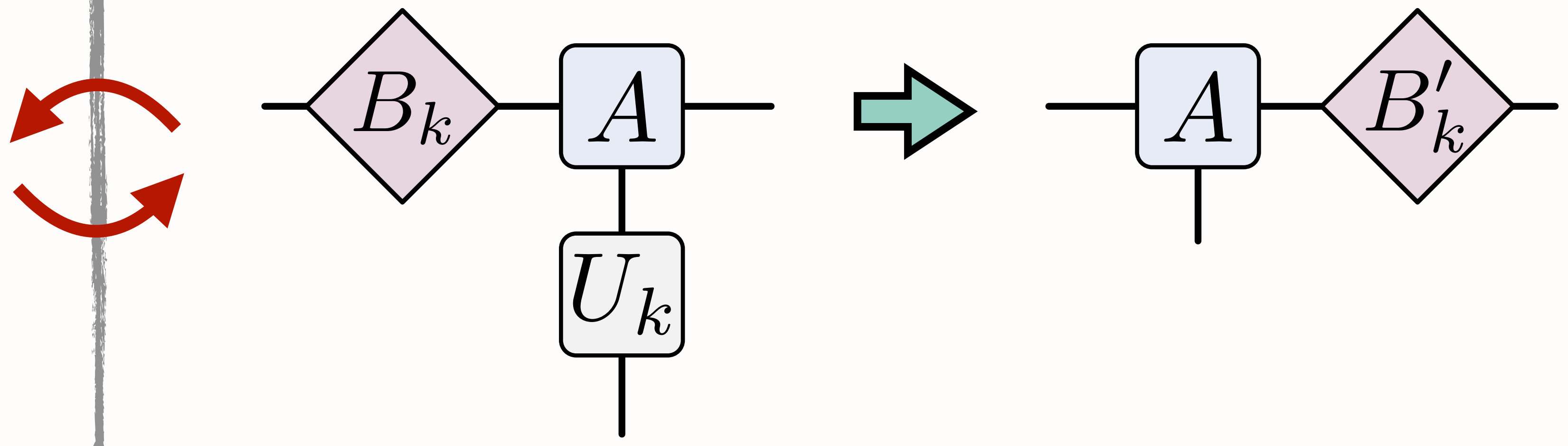

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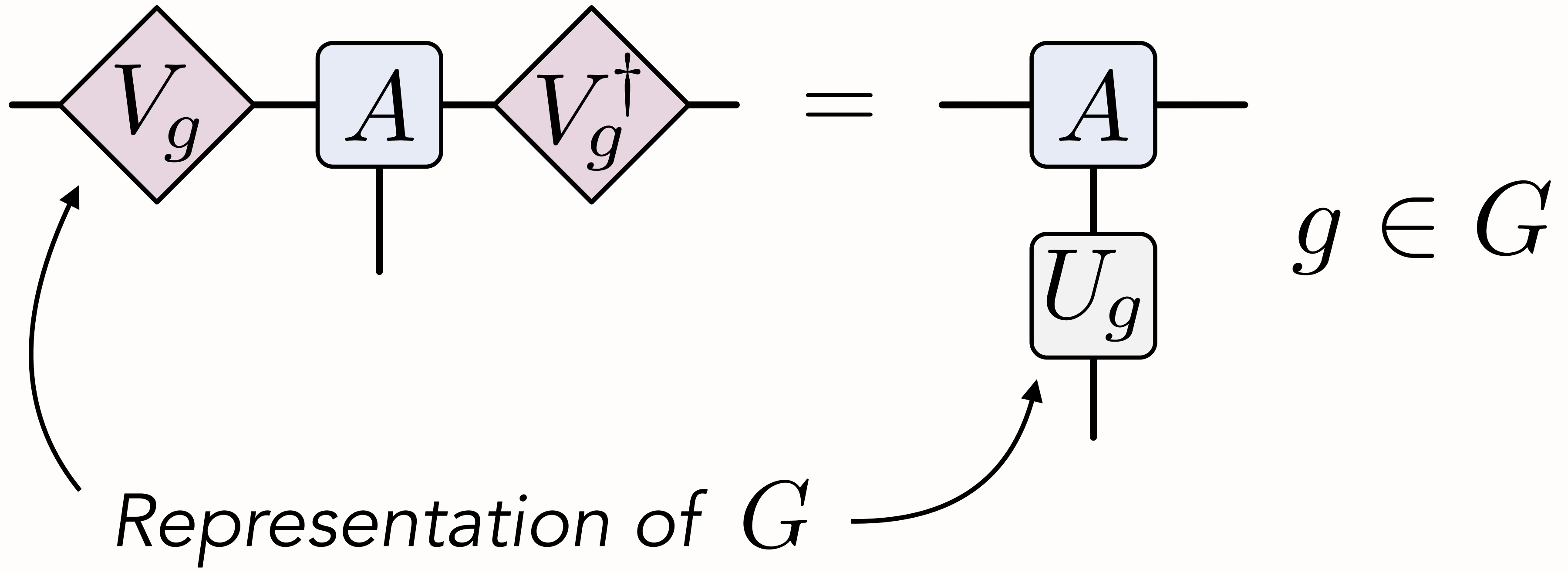


If $\{V_g\}$ form an *irreducible* representation of G

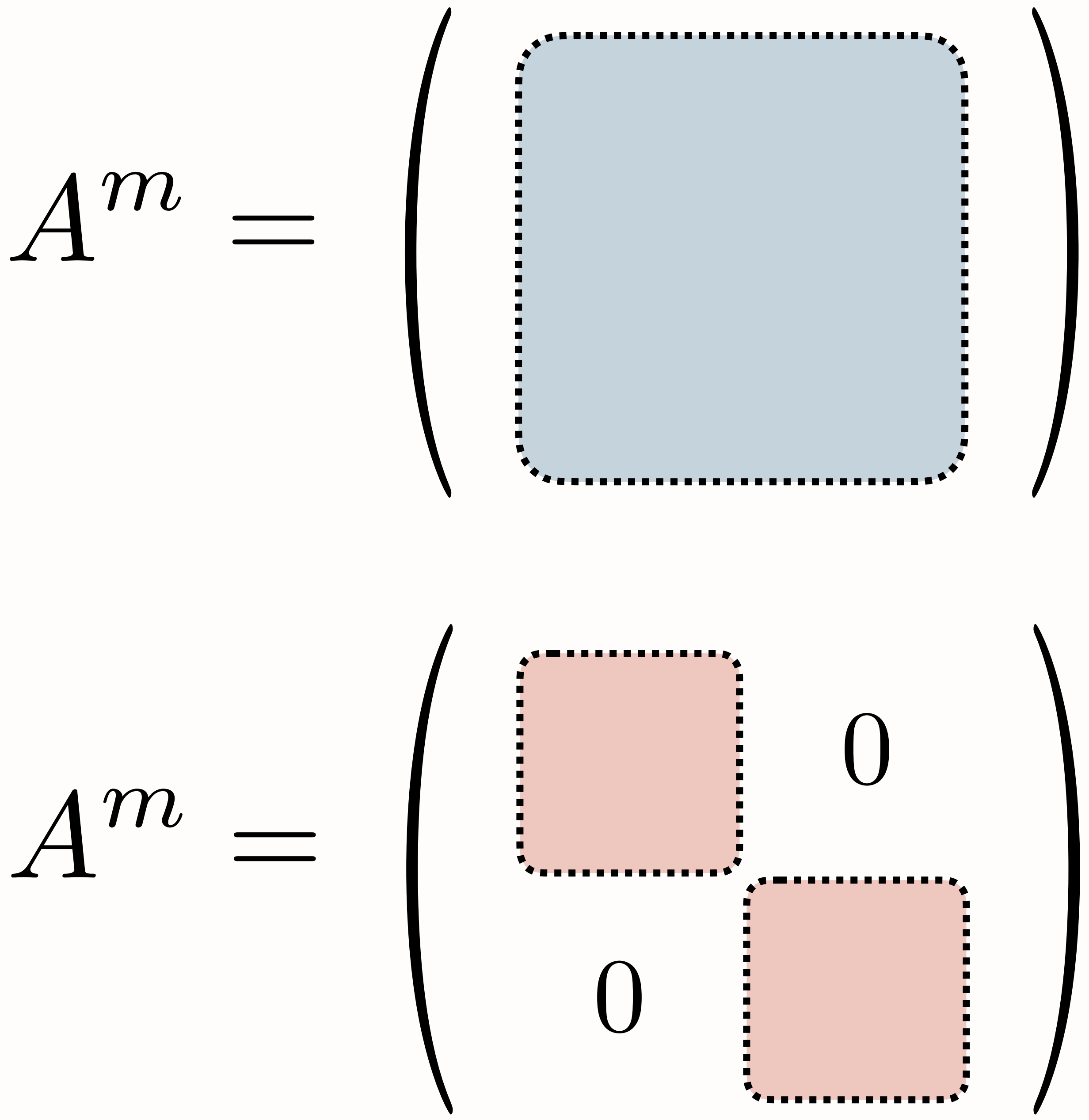
⇩

Can prepare in constant depth

Key takeaway: Symmetry ↔ Measurement basis!

A zoo of MPS: some highlights

- Can prepare both **normal** and **non-normal*** MPS in constant-depth

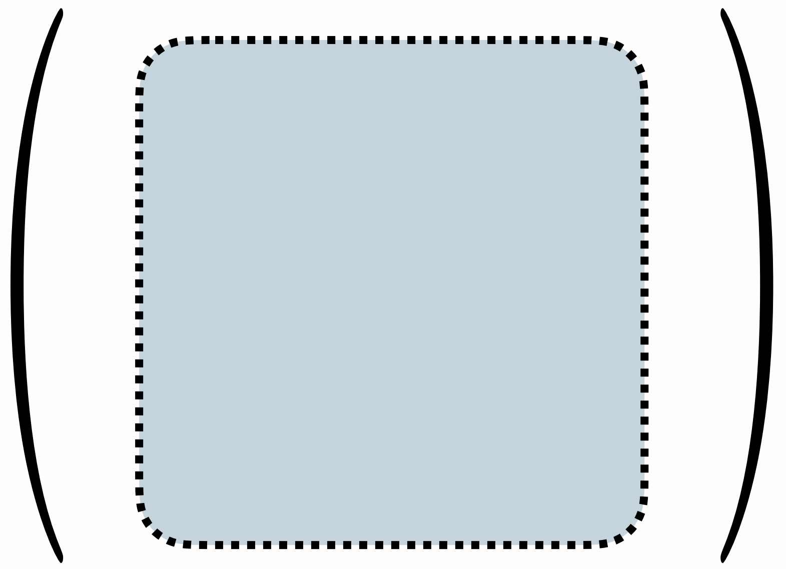


*with two rounds of measurement and feedforward

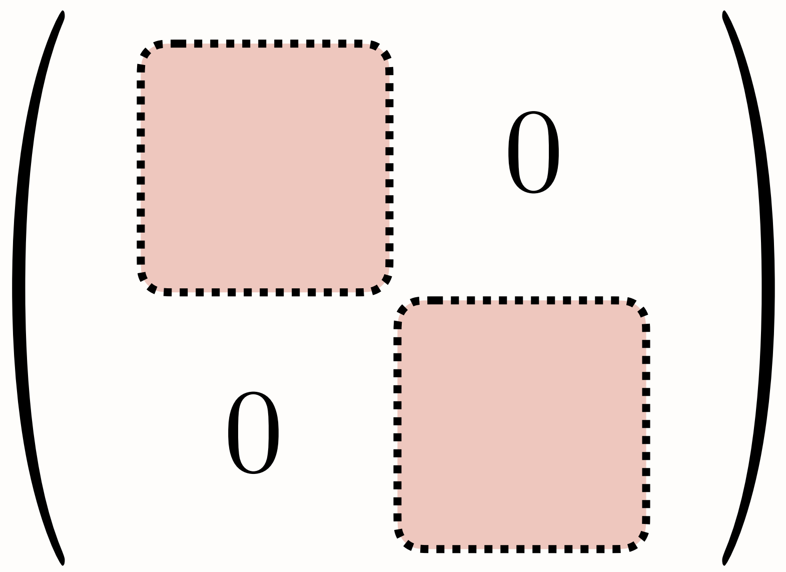
Normal vs. non-normal MPS: Schuch et al., PRB (2011); [1] Malz et al., PRL (2023) [2] Bravyi et al., PRL (2006)

A zoo of MPS: some highlights

- Can prepare both **normal** and **non-normal*** MPS in constant-depth

$A^m =$ 

- Short-range entangled (e.g., AKLT)

$A^m =$ 

*with two rounds of measurement and feedforward

Normal vs. non-normal MPS: Schuch et al., PRB (2011); [1] Malz et al., PRL (2023) [2] Bravyi et al., PRL (2006)

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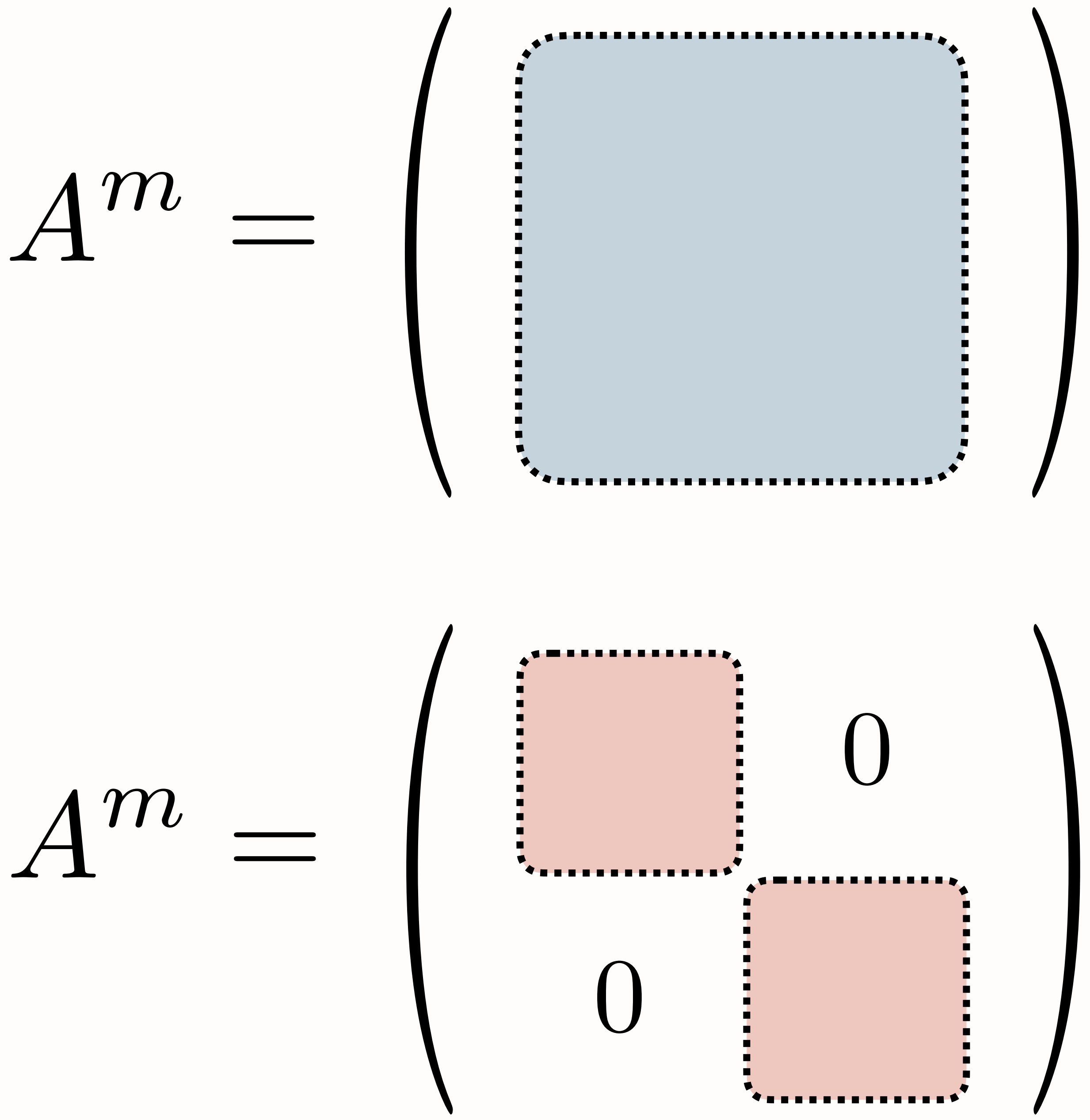

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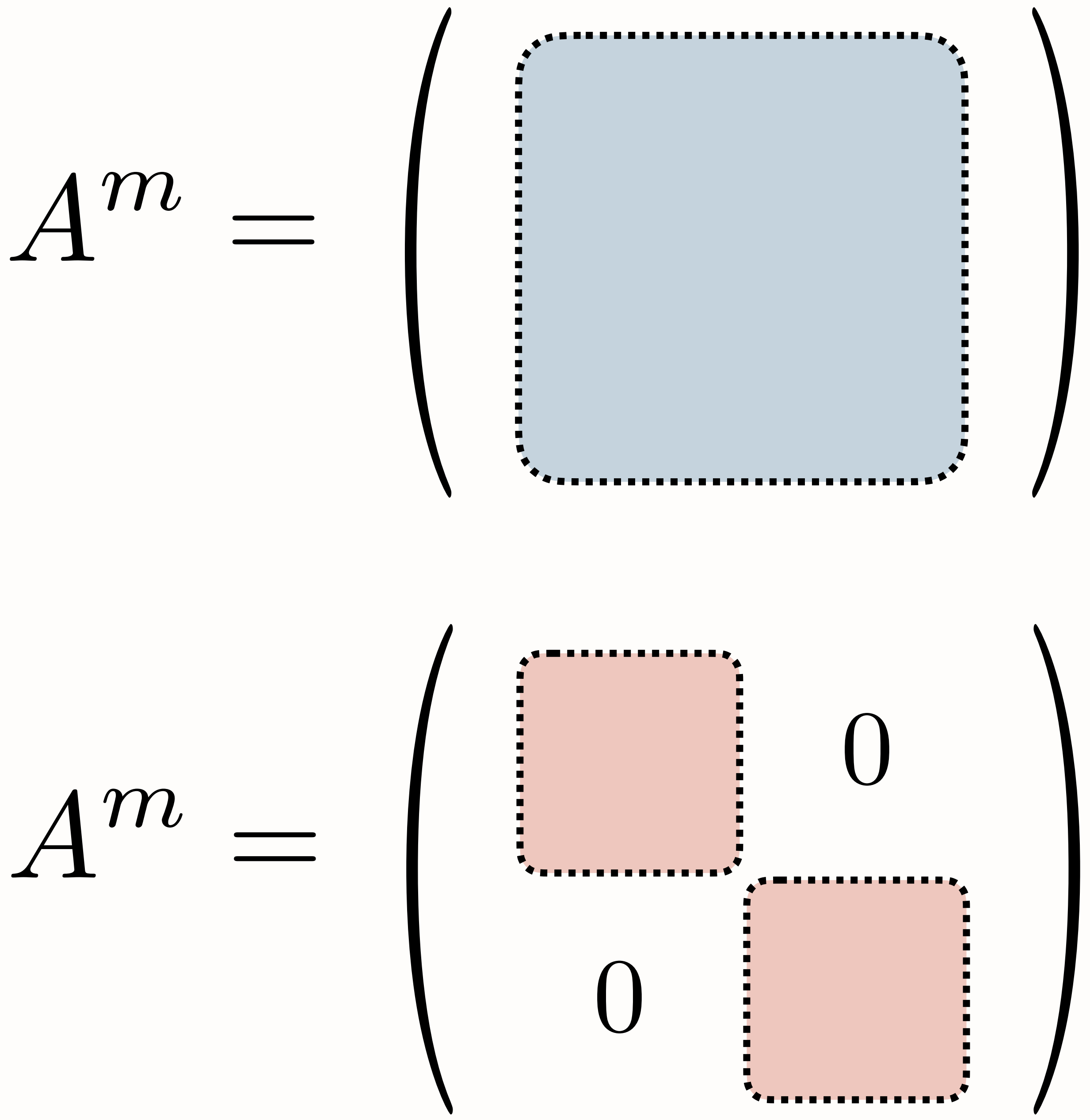

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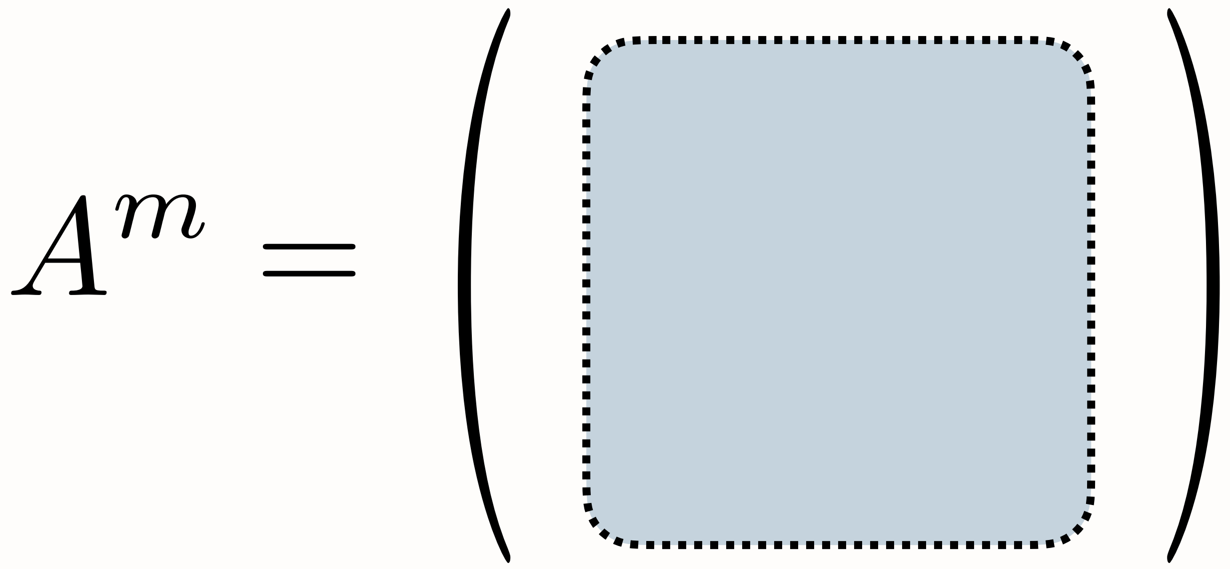

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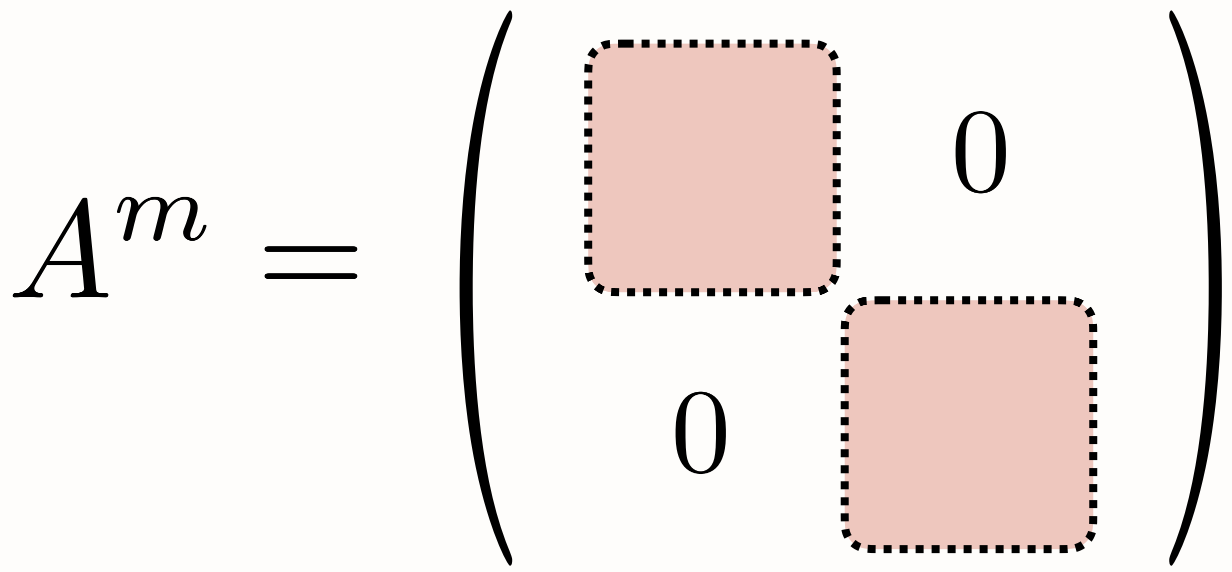

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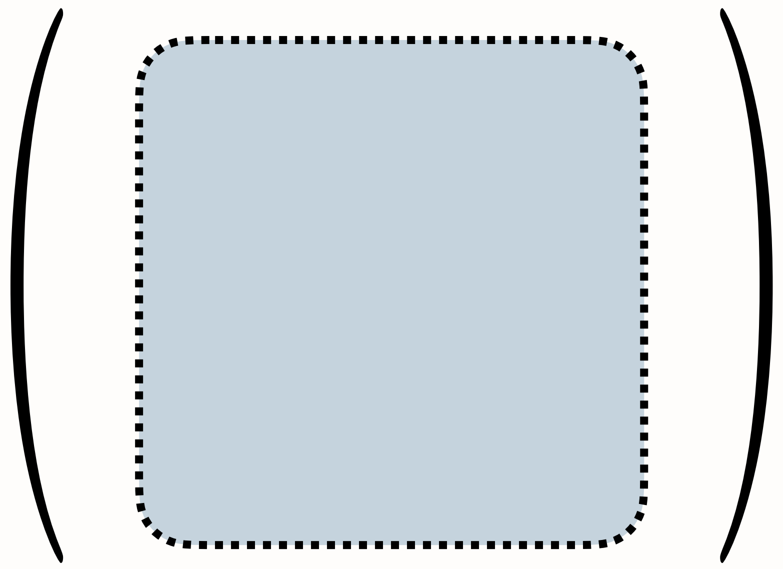
$A^m =$ 

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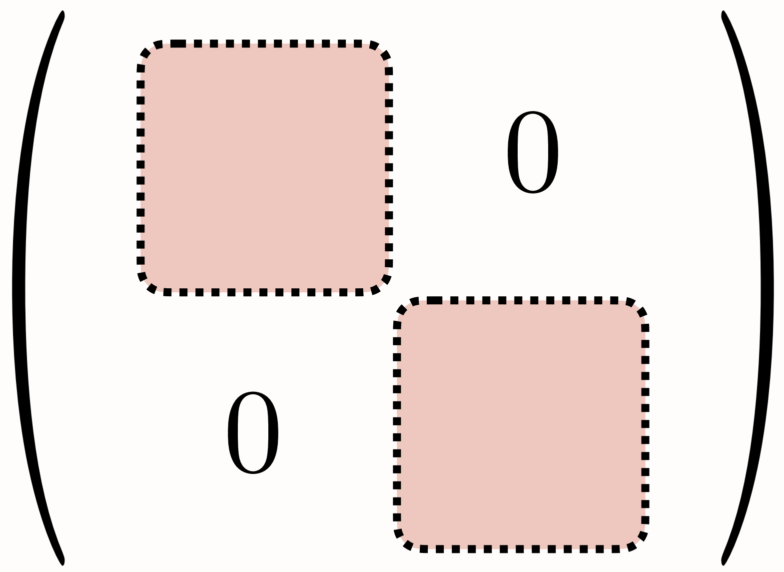
$A^m =$ 

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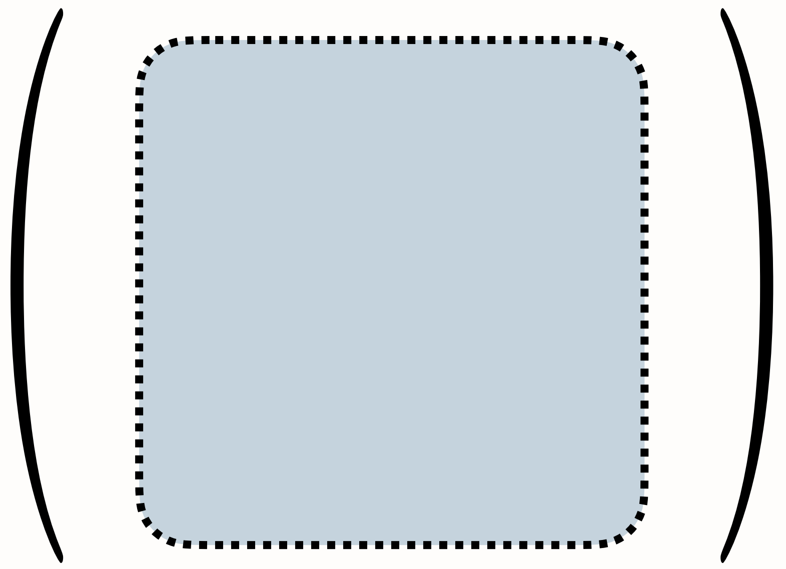

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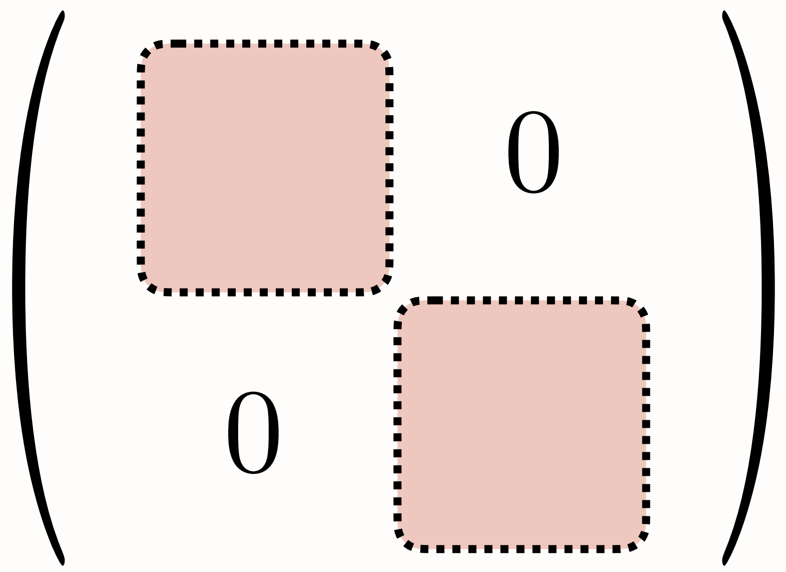

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- Many interesting examples — see Smith *et al.*, PRX Quantum 5, 030344 (2024)

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$$SU(n),\ SO(2\ell+1),\ Sp(2n)$$

Measurement-based quantum computation with qudits [3]

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  - Sampling random MPS (highly magical [4]), or from a SPT phase

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I. Introduction to MPS

1. What are they?

2. Why prepare them?

II. Techniques for preparing MPS on quantum processors

1. Unitary preparation — various flavors and current literature

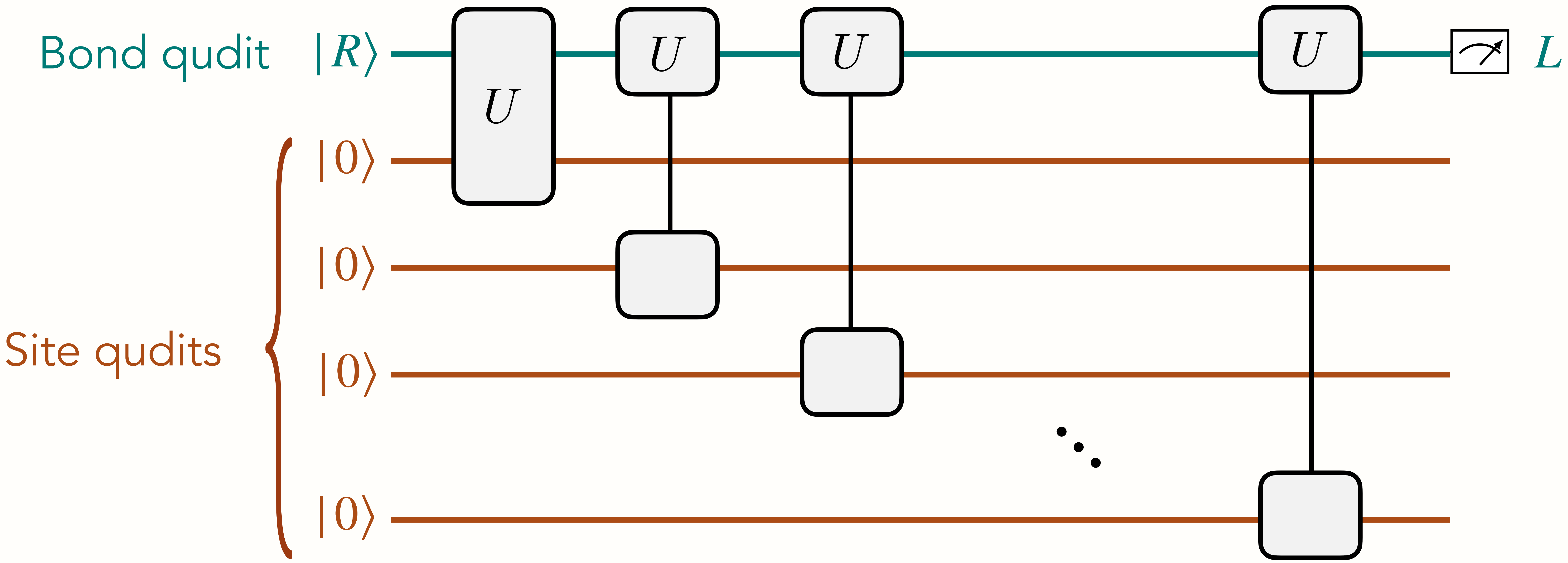
2. Adaptive preparation (temporal compression)

3. Bonus: Holographic preparation (spatial compression)

“Holographic” preparation: high-level idea

Foss-Feig et al., Physical Review Research (2021); Foss-Feig et al, Phys. Rev. Lett. (2021)

$$|\Psi\rangle = \sum_{\vec{m}} \langle L| A^{m_1} A^{m_2} A^{m_3} \dots A^{m_N} |R\rangle \, |m_1 m_2 m_3 \dots m_N\rangle$$



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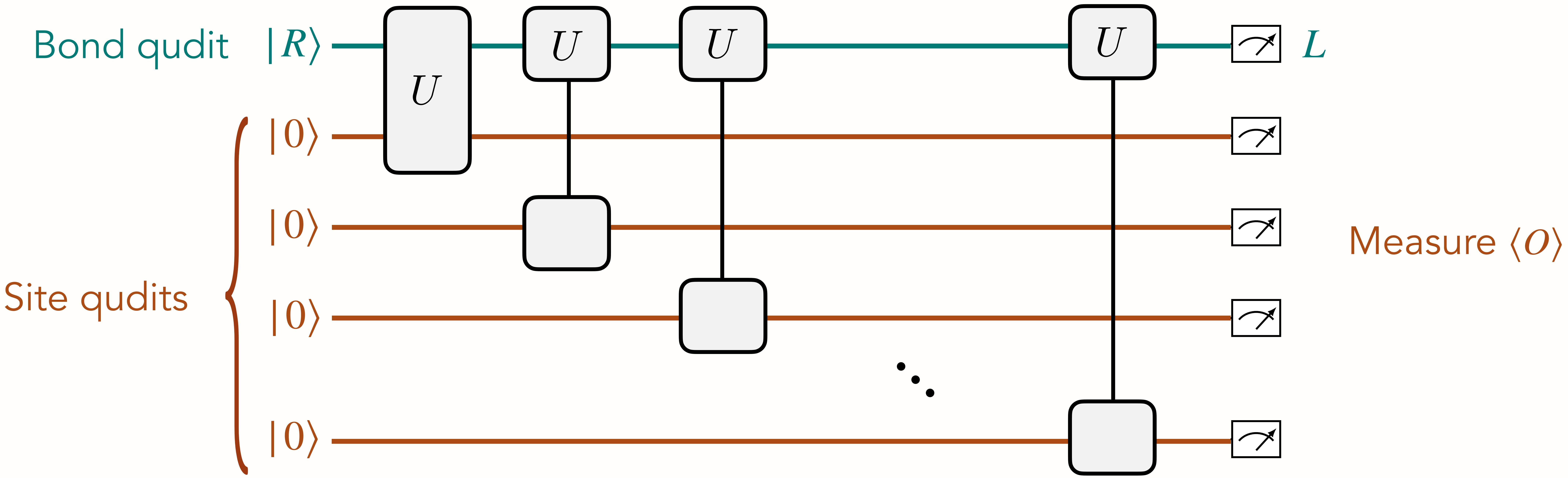

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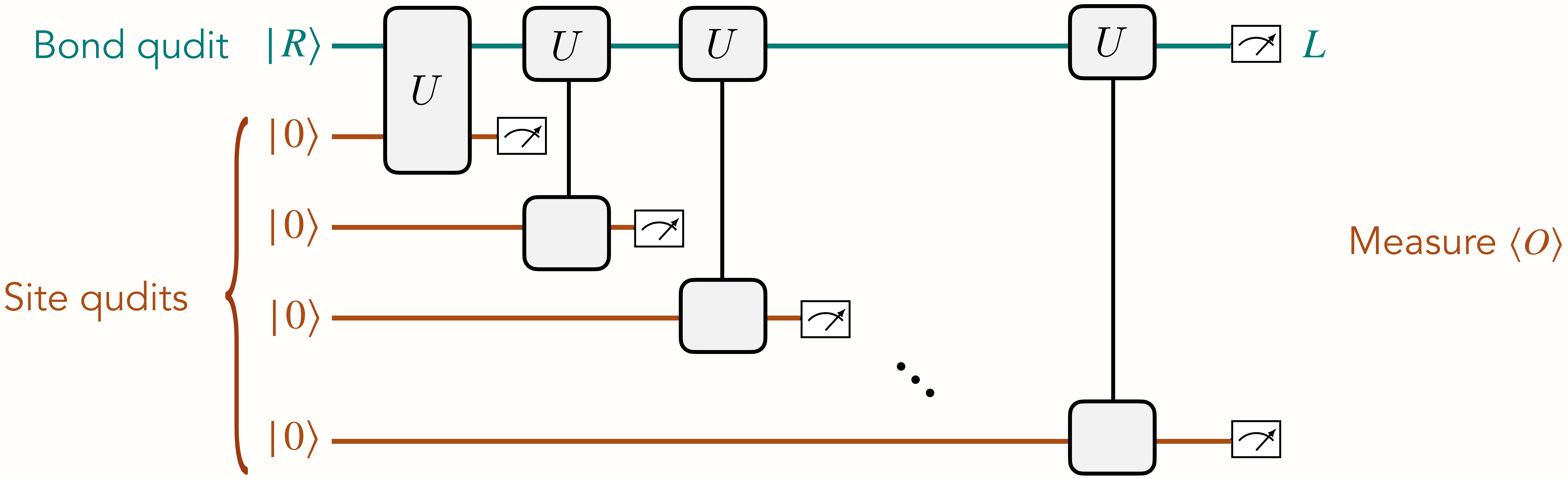

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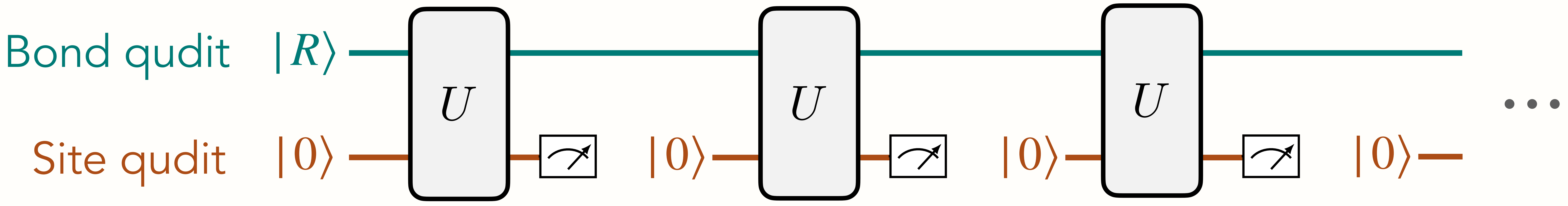

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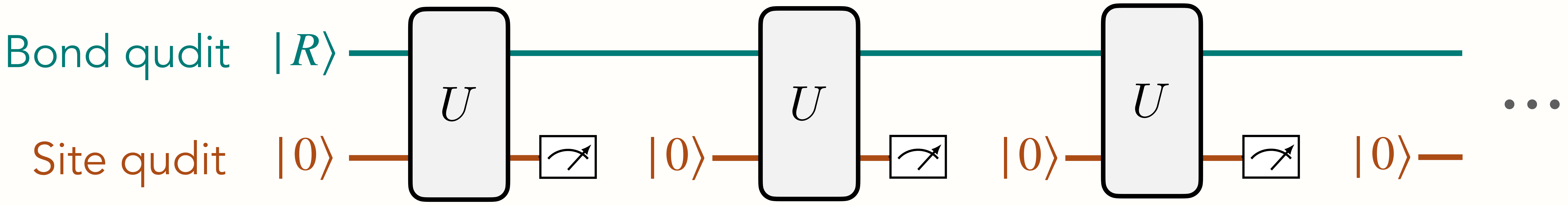
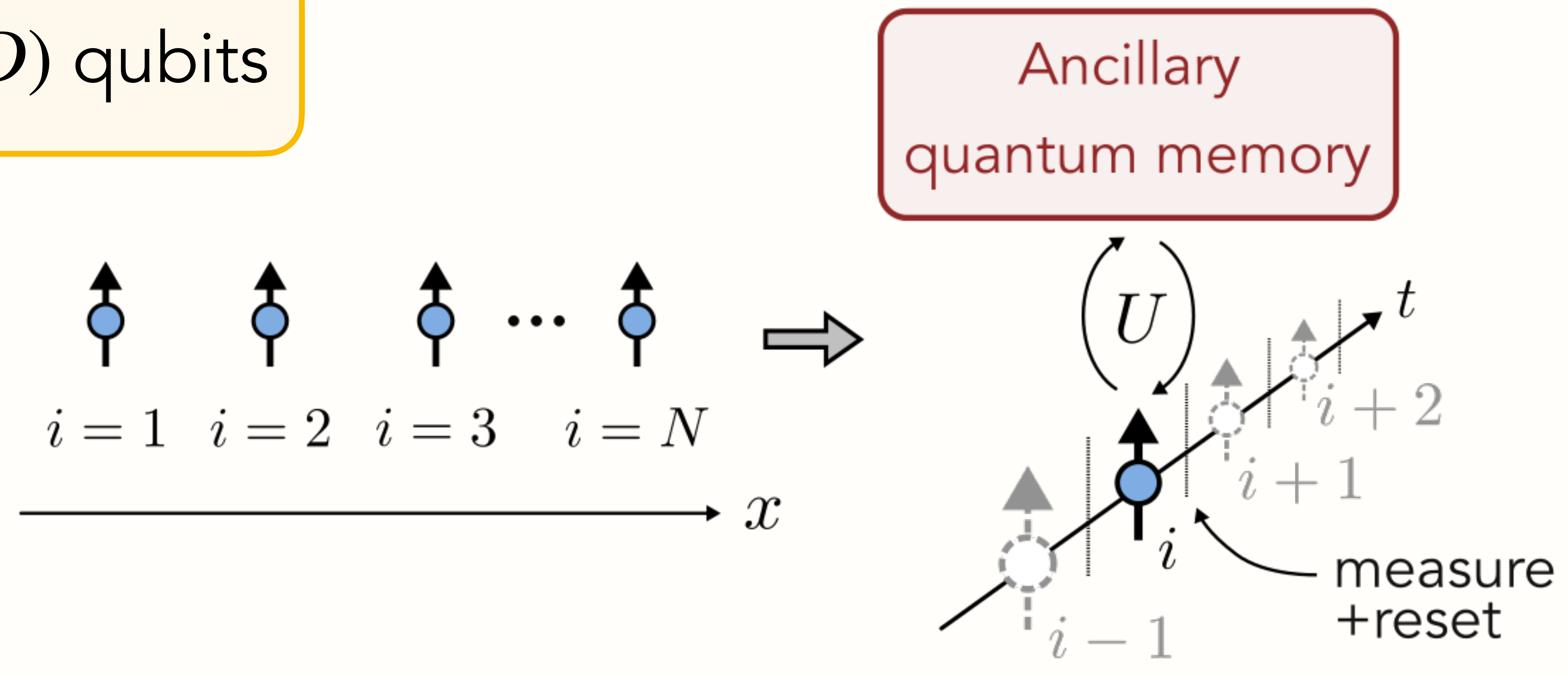


Spatial resources: $\log_2(dD)$ qubits



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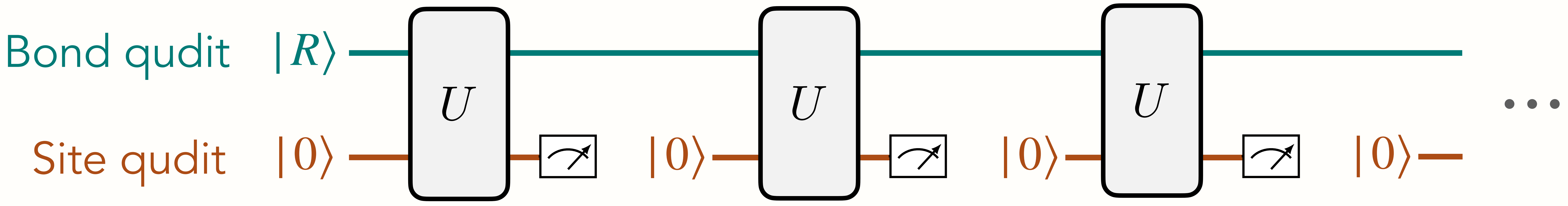
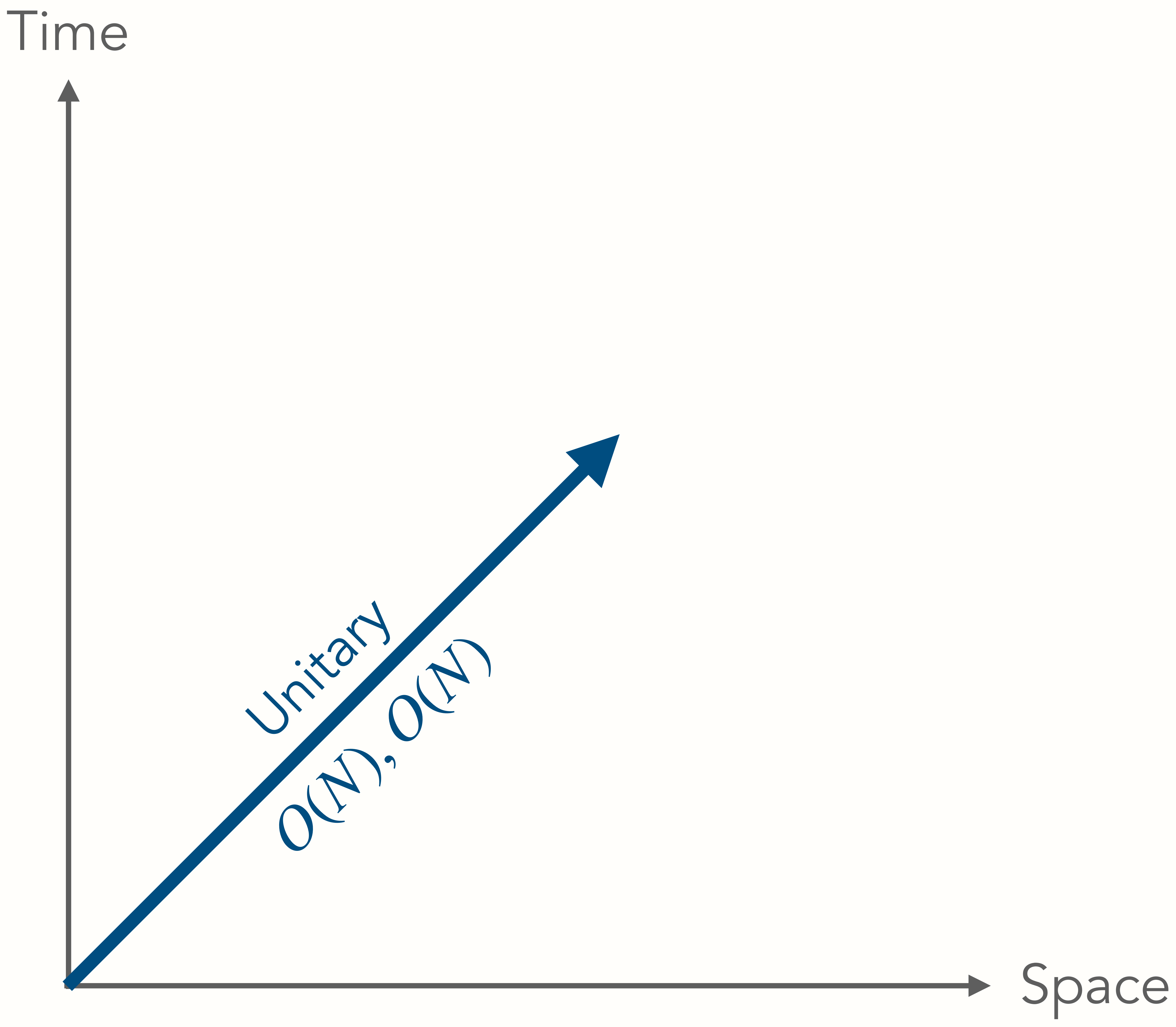
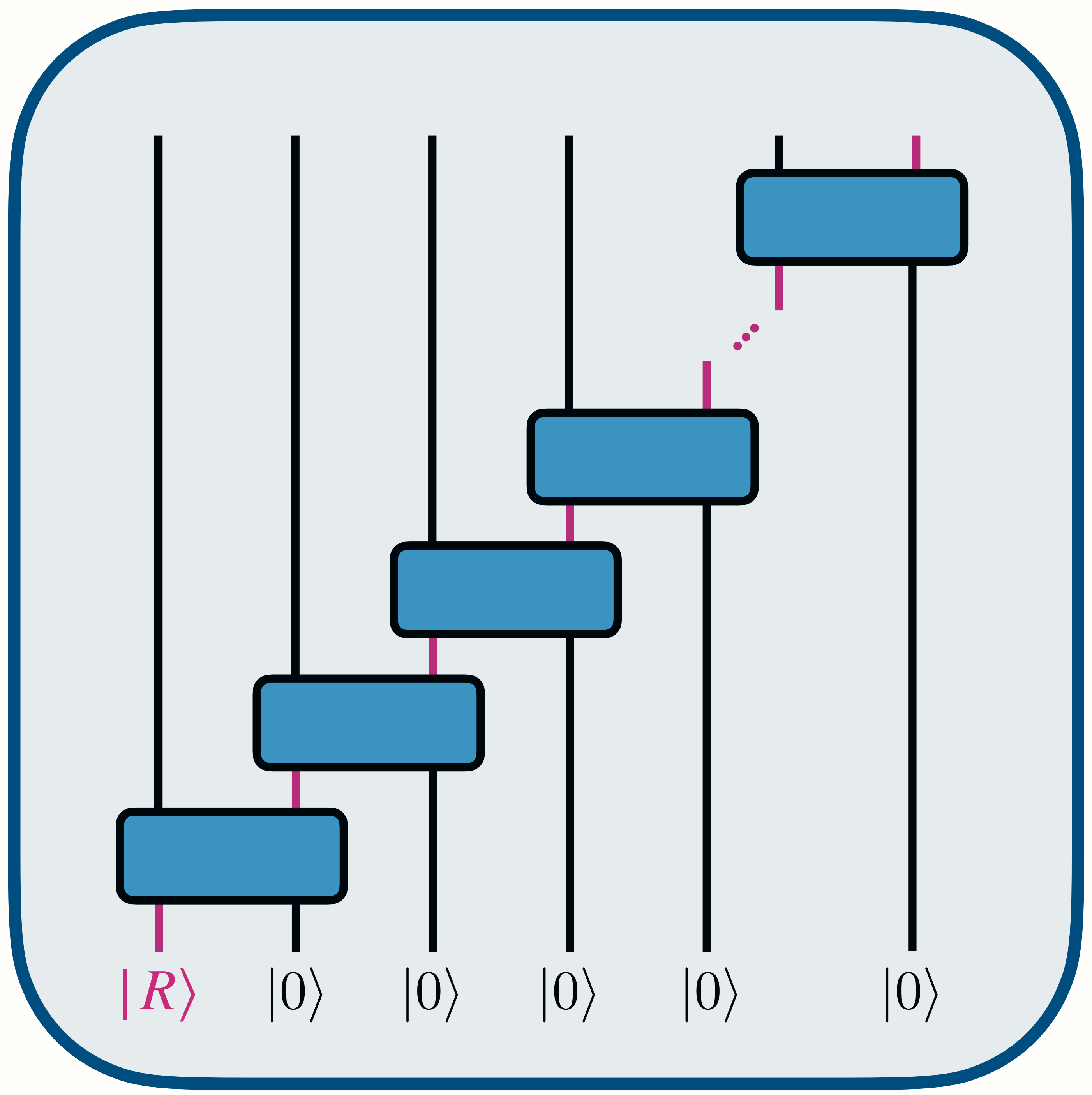


Spatial resources: $\log_2(dD)$ qubits

- holoVQE: $U \to U(\theta)$
- holoQUADs: time dynamics [Chertkov et al., Nature Physics (2022)]
- Higher dimensional (isometric) tensor networks [Liu et al., Phys. Rev. Research (2024)]

Closing thoughts: a spacetime picture





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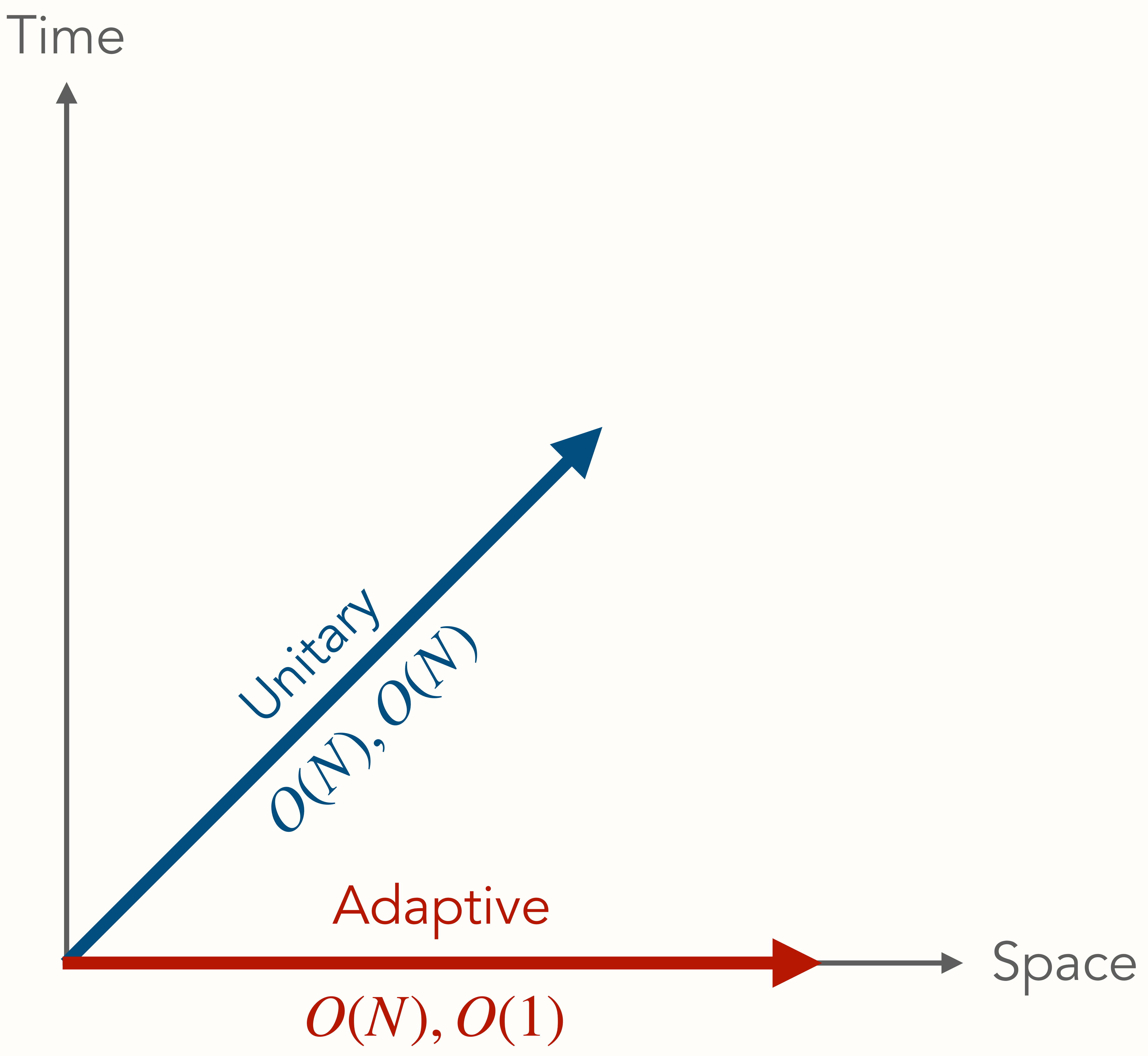

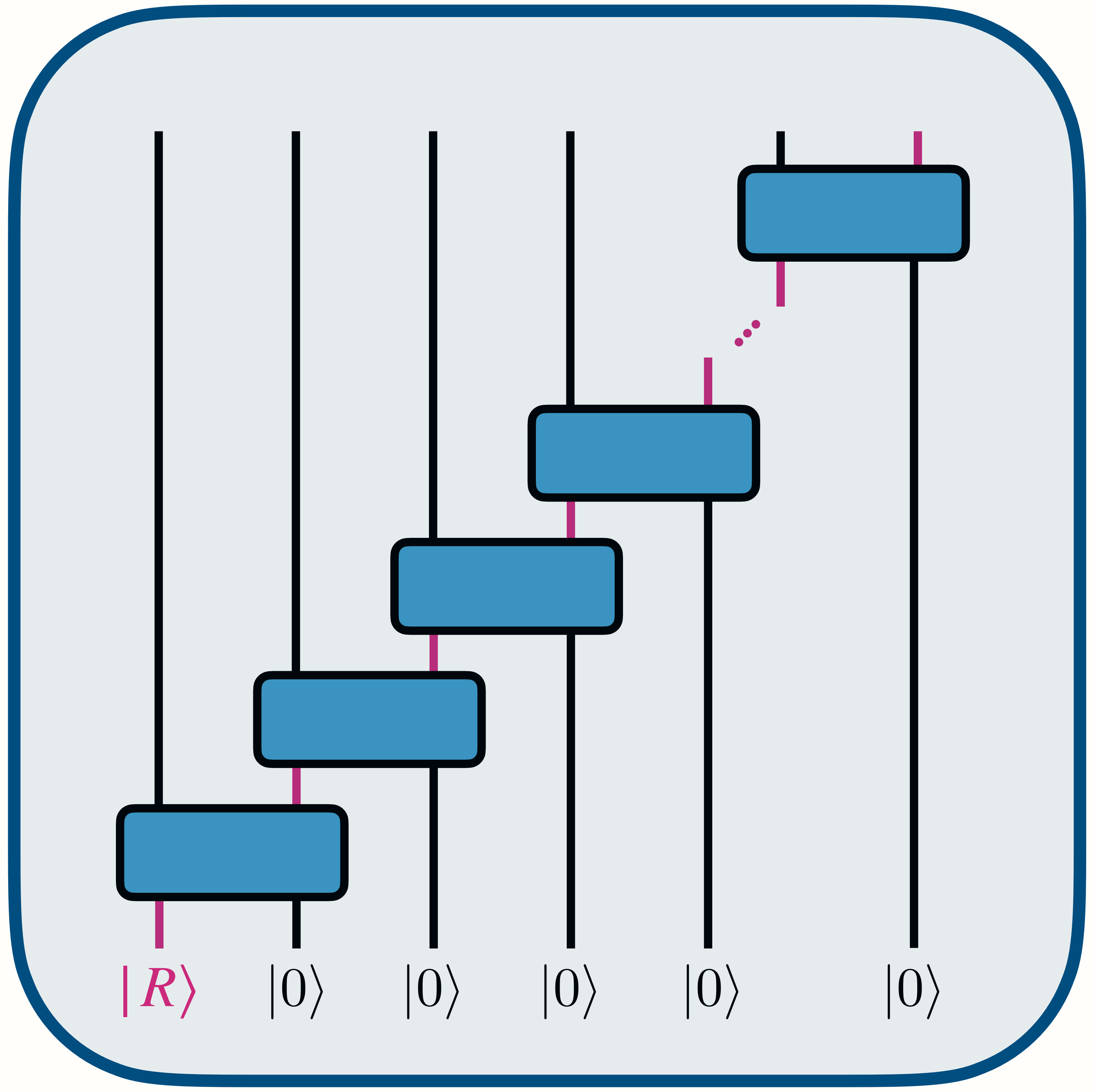

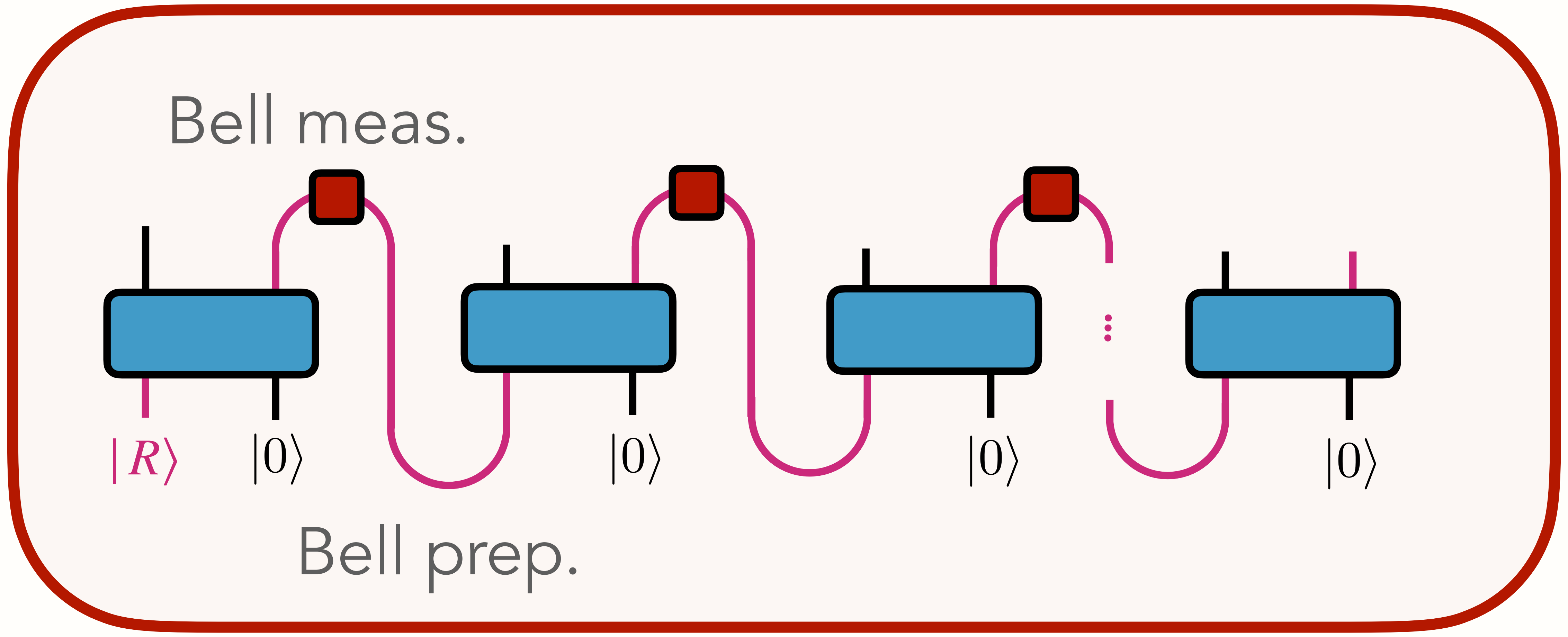

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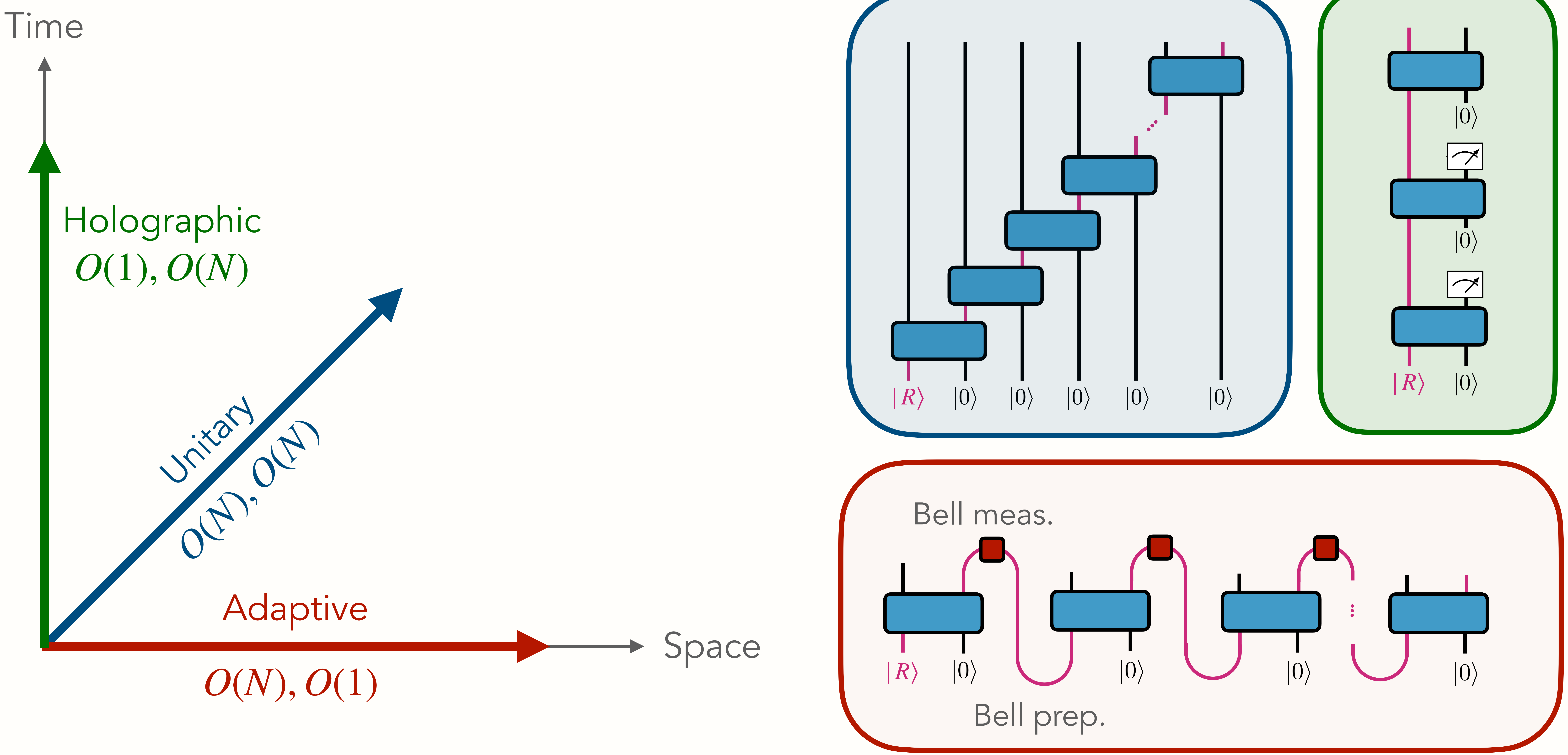

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Ella Crane
(MIT)


Nathan Wiebe
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Yale YQ C²QA Brookhaven National Laboratory IBM Quantum

Smith, Crane, Wiebe and Girvin, PRX Quantum 4, 020315 (2023)

Smith, Khan, Clark, Wei, and Girvin, PRX Quantum 5, 030344 (2024)